The Bernoulli equation and compressible flow theories

Walter E. Frick
US Environmental Protection Agency, 960 College Station Rd, Athens, GA30605-2700

Abstract. The incompressible Bernoulli equation is an analytical relationship between pressure, kinetic energy, and potential energy. As perhaps the simplest and most useful statement for describing laminar flow, it buttresses numerous incompressible flow models that have been developed to model turbulent flow. In a re-examination of the derivation, Frick (2001) shows that the Theorem of the Mean for Integrals, not the assumption of absolute incompressibility, is the underlying justification for the simplification that makes it possible to derive the Bernoulli equation from the Euler equation. In other words, the assumption of incompressibility is unnecessary and the equation actually represents a compressible relationship. While this finding supports the expansive turbulence model proposed by Frick et al. (2001), a closer look at their derivation of flow between two disk suggests that the underlying approximations are not entirely consistent with the theory. In other words, the physics of the expansive turbulence model proposes that the divergence of the velocity is positive, which is not what their solution shows.