A Stochastic Differential Equation Approach for Modeling DNAPL Flow in Heterogeneous Porous Media

D. W. Dean[1] and T. F. Russell
Department of Mathematics, University of Colorado at Denver, Denver, Colorado

T. H. Illangasekare
Center for Experimental Study of Subsurface Environmental Processes (CESEP), Colorado School of Mines, Golden, Colorado

Abstract. This research explores the use of stochastic differential equations (SDE's) to model multiphase flow in heterogeneous aquifers, specifically the flow of DNAPL's in saturated soils. The approach given results in a nonlinear SDE describing the position of the non-wetting phase fluid particle. A fundamental assumption used in the model formulation is that the flow of fluid particles is described by a stochastic process and that the positions of the fluid particles over time are governed by the law of the process. The nonlinearity in the SDE arises because both the drift and diffusion coefficients depend on the volumetric fraction of the phase which, in turn, depends on the position of the particles. Darcy's law and the continuity equation are then used to derive a Fokker-Planck equation from these expressions. The Ito calculus is then applied to derive a SDE for the non-wetting phase. Stochastic models like these are typically used to model dispersion processes. Such models, in their usual form, cannot represent multiphase barrier effects. These effects occur at the interface between adjacent materials with different permeabilities. The behavior of the plume at an interface depends on the pressure-saturation curves involved in forming the interface. In the model, the control of the flow of DNAPL particles across the interface is accomplished using a jump term which derives from the Ito formula. The jump term is based on capillary diffusivity and the pressure-saturation curves of the sands forming the interface. Computational examples are given.

[1] University of Colorado at Denver and Health Sciences Center
Department of Mathematics
Campus Box 170, P.O. Box 173364
Denver, CO 80217-3364
Tel: (303) 556-6267
Email: ddean@math.cudenver.edu