Application of the fuzzy theory in a reservoir operation model, to study the behavior of the regularized flow

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Abstract - This work concerns with the application of Fuzzy Theory in the hydrologic system as a reservoir, to evaluate its forecast capacity, in the calculation of the risk of collapse of systems composed by this type of body of water. In the development of the research, a methodology, transforming the equations of the hydrologic balance, in fuzzy equations, was applied. The flow and the income net were calculated, for different sceneries, as membership functions, where those control variables, with larger pertinence degree, were analyzed. The results showed that the methodology fuzzy could come as an important alternative in the calculation of the risk of collapse of hydrologic systems, as well as, it can, equally, come as a good alternative in the determination of the sustainability of water, in areas with high vulnerability degree, as it happens in semi-arid regions.

Keywords: Fuzzy Set Theory; Reservoirs Operation; Fuzzy Models.

1 Introduction

The irregularities of the rains, presents in semi-arid areas of the planet earth, constitute in a serious problem that has been worrying authorities, researchers, engineers and technicians, considering its consequences, not only in the economic planning, as in the social planning of these areas. As it is known, it is very uncertain the presence of significant investments in areas that don't offer concrete warranties of water readiness for of long period. In this context, the Brazilian Northeast, a semi-arid area, with serious irregularities in their hydrological subjects, has been having some difficulties in its process of reception of resources to sustain its development.

On the other hand, the study of reservoirs operation involves multidisciplinary processes and the necessity of a solid database, where the hydrological uncertainties can be appropriately acted so that the risk of those systems fail can be evaluated. Usually, in the evaluation of the risk of fail of any hydrological model, the application of the Probabilities Theory can play important part in this class of study. However, this theory, as it was said previously, needs databases with historical series, for long periods, describing the behavior of a watershed. These databases are not found with so much frequency in underdeveloped countries.

Actually, a new theory is appearing, with similar characteristics the those found in the confidence intervals, to evaluate risk analysis in the field of the engineering. This theory, call of Fuzzy Set Theory, is based in the membership function that applies to a certain numeric group related with a certain process. The great advantage of this theory is in the fact that it does not need a great database to present good results. The Fuzzy Set Theory ap-

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peared for the first time in the sixties and, starting from this date, its application is feeling in the most varied fields of the science, besides in the field of water resources.

This research applied the Fuzzy Set Theory in a simple modeling of reservoirs operation, where the mass balance consists, just, in controlling the regularized flow, in function of the evaporation and of the time of emptying, starting from a volume stored in this body of water, to test the efficiency in a possible evaluation of the risk of fail in this system. The proposal idea is of making calculations, starting from some hydrological parameters, presented as membership functions, the regularized flow and the income flow, for different proposed sceneries.

The results showed that the Fuzzy Set Theory can come as an important alternative, in the studies of reservoirs operations, for the determination and evaluation of the risk of fail for a hydrological system with high vulnerability degree.

2 Methodology

Equation for Time to Empty the Reservoir

Using as base the Method of the Triangular Regularization Diagram together with the mathematical program Maple 9.5, Campos (2006) developed an equation to esteem the time to empty the reservoir. For this formulation, the morphology of the hydraulic basin was represented through the equations:

$$V(h) = \alpha h^3 \tag{1}$$

$$A(h) = 3\alpha h^2 \tag{2}$$

Where:
V(h): it Represents the volume of the reservoir with height h;
A(h): it Represents the area of the lake of the reservoir with height h;
α : factor in way of the reservoir;

The variation of the volume of the reservoir can be described by the equation:

$$dV = \alpha h^3 - \alpha\left(h - dh\right)^3 \tag{3}$$

Where dh is the infinitesimal representation of the depth.
Making an expansion of the equation 3, we have:

$$dV = \alpha h^3 - \left[\alpha h^3 - 3\alpha h^2 dh + 3\alpha h dh^2 - \alpha dh^3\right] \tag{4}$$

Knowing that dh is a very small increment, their potencies above 2 can be despised. So that the equation assumes the form:

$$dV = 3\alpha h^2 dh \tag{5}$$

As for the variation of the volume of the reservoir, this can be dear through the equation:

$$dV = qdt + e3\alpha h^2 dh \tag{6}$$

where:

q: represents the volume that comes out of the reservoir;
e: represents the evaporated intensity;
t: the time interval.

Making the equality of the equations (5) and (6) we obtain:

$$dt = \left[\frac{3\alpha h^2}{q + 3e\alpha h^2} \right] dh \tag{7}$$

That it is a differential equation of first order and second degree that it relates the flow regularized with the time of emptying. The time to empty the reservoir is obtained through the resolution of the equation (7):

$$te = \frac{h}{e} - \frac{q\sqrt{3} \cdot \arctan\left(\frac{e\alpha h\sqrt{3}}{\sqrt{qe\alpha}} \right)}{3e\sqrt{qe\alpha}} \tag{8}$$

where:

te: represents the time to empty the reservoir;
h: depth of the reservoir;
q: the retreat flow;
e: the evaporation intensity;
α: it is the factor in way of the reservoir.

The accumulation efficiency, it is evaluated by the relationship among the volume used (q.te) and the accumulated volume in the end of the humid station (K), calculated through the equation,

$$\eta = \frac{q.te}{K} \tag{9}$$

where:
η: it is the accumulation efficiency;
q: the used volume;
te: the time of emptying;
K: the accumulated volume in the end of the humid station.

Fuzzy Set Theory

According to Saavedra (2003) the conventional logic treats the information in a binary way, classifying them as true or false. Maybe the definition of those two states of the information, in some cases, be enough. However, many human experiences need an including manipulation than the simple treatment of false or true, yes or no, certain or wrong. It is in this context that the logic fuzzy if it turns an appropriate tool to treat vague and uncertain information, in general described in a natural language (LIMA, 2002).

An eminent factor of that theory is its capacity to capture intuitive concepts, besides considering psychological aspects used by the human beings in its usual reasoning, avoiding that its representation is plastered by traditional models (OLIVEIRA, 1999).

It was in 1965, thinking about attributing meanings the linguistic terms that mathematician Lofti Zadeh, introduced the concept of groups fuzzy. Through such groups, it would be possible to store no necessary data in computers, to generate answers based on information vague or ambiguous, in processes similar to the human reasoning. In this logic, mathematical models are used to map subjective variables, as cold, pleasant and hot, for concrete values that can be manipulated mathematically.

In agreement with Ganoulis (1994), the central concept of the theory of the numbers Fuzzy bases on the existence of a membership function to represent numerically the degree through which certain element belongs to a group. Like this, according to Zadeh (1965) a group fuzzy is characterized by a membership function that will map the elements of a certain domain for a real number belonging to the interval [0,1].

Usually, a membership function is in the form $\tilde{A}: X \rightarrow [0,1]$. like this being, any function acted like this can be associated to a group fuzzy, depending on the concepts and of the properties that one want to act, being considered, still, the context in which the group is inserted. A group fuzzy is a group of orderly pairs where the first element is $x \in X$ and the second is the membership function Ã (x) that maps x in the interval [0,1]. Like this, the representation of a group fuzzy is defined mathematically for:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) x \in X; \mu_{\tilde{A}}(x) \in \left[0,1 \right] \right\} \qquad (10)$$

where:

$\mu_{\tilde{A}}(x)$ is the pertinence degree of in the group.

Several types of membership functions as, for instance: triangular, trapezoidal, exponential, and Gaussian. The functions more used are the trapezoidal and the triangular ones.

Fuzzy Equations Model

The fuzzy model developed for this research has for base to treat the time of emptying of the reservoir and its accumulation efficiency as numbers fuzzy, acted by their membership functions, defined in the interval [0,1].

A membership function is one that represents the level of pertinence of the parameters, in a certain much defined physical process. This way, as bigger is the degree of pertinence of this variable in the context, the bigger will be the value of this function. In the development of the fuzzy model it is necessary to redefine the emptying of the reservoir and of the accumulation efficiency equations as membership functions.

That is done transforming each control variable, in a fuzzy variable. For instance, the time to empty the reservoir, calculated through the equation (8), changes from a simple function of the domain in real numbers, to a membership function. Therefore, the following formulations are:

Equation of Time to Empty the Reservoir

$$\tilde{te} = \frac{h}{\tilde{e}} - \frac{\tilde{q}\sqrt{3}.\arctan\left(\dfrac{\tilde{e}\alpha h\sqrt{3}}{\sqrt{\tilde{q}\tilde{e}\alpha}}\right)}{3\tilde{e}\sqrt{\tilde{q}\tilde{e}\alpha}} \tag{11}$$

Equation of Efficiency of Accumulation

$$\tilde{\eta} = \frac{\tilde{q}\cdot\tilde{te}}{\tilde{K}} \tag{12}$$

where:
$\tilde{e}$: Membership function for the evaporation;
α : Form factor of the reservoir;
$\tilde{te}$: Membership function for the time of emptying;
h: depth of the reservoir;
$\tilde{q}$: Membership function for the flow;
$\tilde{\eta}$: Membership function for the accumulation efficiency;
$\tilde{K}$: Membership function for the capacity of the reservoir.

The solution of this group of equations allows determining the dependent variables in the membership form. Those functions are calculated along several depths of the reservoir for different sceneries of simulations. In other words, there is a membership function for the time of emptying, for the evaporation, for the capacity of the reservoir. This implicates to say that the model allows evaluating the behavior of the regularized flow and the efficiency, for different depths and under different sceneries.

In scenery applied, it was obtained membership functions for the regularized flow and the efficiency, along different heights, considering in simulations that the evaporation was maintained equal 191mm/mês, the form factor of the reservoir α=1000. In this scenery two cases were analyzed, with two different membership functions for the time to empty the reservoir. In this way, they were used the functions te=[3;4;6] month and te=[3;5;6] month.

3 Results

In order to analysis of the problem in study, where it intended to verify the behavior of the regularized flow and of the efficiency of the reservoir of small load, inside of a fuzzy methodology, a group of simulations was accomplished for that scenery. In this scenery, they were obtained the membership functions of the flow and of the efficiency by running the computational program, where was considered the evaporation constant and equal to 191 mm a month. The form factor of the reservoir =1000 and for the time to empty the reservoir, two membership functions were adopted te = [3;4;6] month and te = [3;5;6] month.

The Figures 1 and 2 show the behavior of the efficiency and of the regularized flow as membership functions, taking the heights as references.

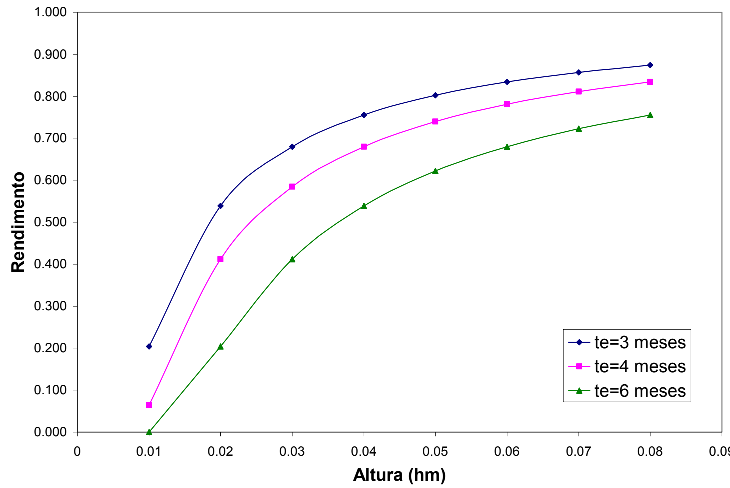

Figure 1 - Behavior of the efficiency with the height, for te=[3;4;6] month.

Analyzing the Figure 1, it can be verified that the membership function representing the efficiency grows with the heights until reaching a level where that growth is little sensitive. In this case, for a depth around 7 or 8 m, its variation is very small, differently that it happens for the most shallow depths, showing that the efficiency very sensitive for low depths.

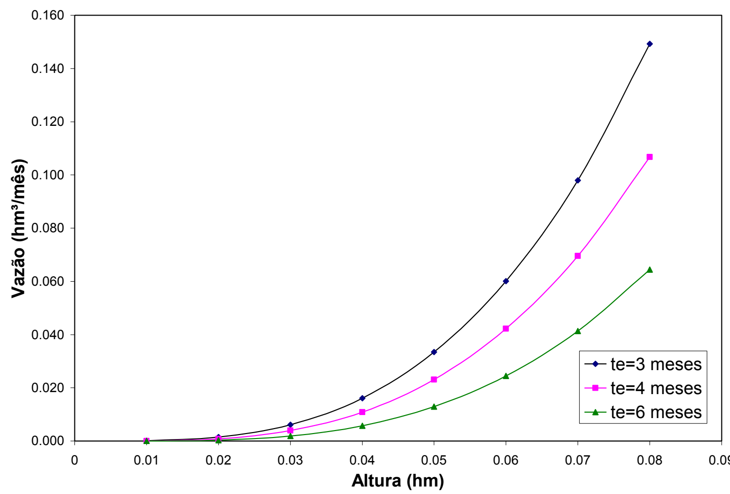

Figure 2 - Behavior of the flow with the height, for te=[3;4;6] month.

Inverse situation happens with the flow, Figure 2, because for their high depths values the regularized grows quickly. Another observation in these figures is the behavior of the line that represents the largest value of membership level of the efficiency. This is closer to the membership with low efficiency value than the membership with upper efficiency value, showing that the membership functions are asymmetric with respect to the efficiency value with great pertinence value. For the membership function of the regularized flow, that asymmetry is not so evident.

The Figures 3 and 4 show the behavior of the efficiency and of the regularized flow for a time to empty the reservoir te=[3,5,6].

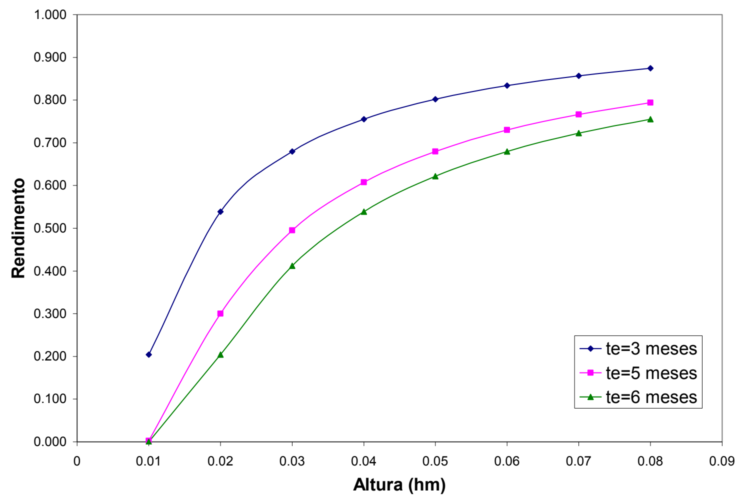

Figure 3 - Behavior of the efficiency with the height, for te=[3;5;6] month.

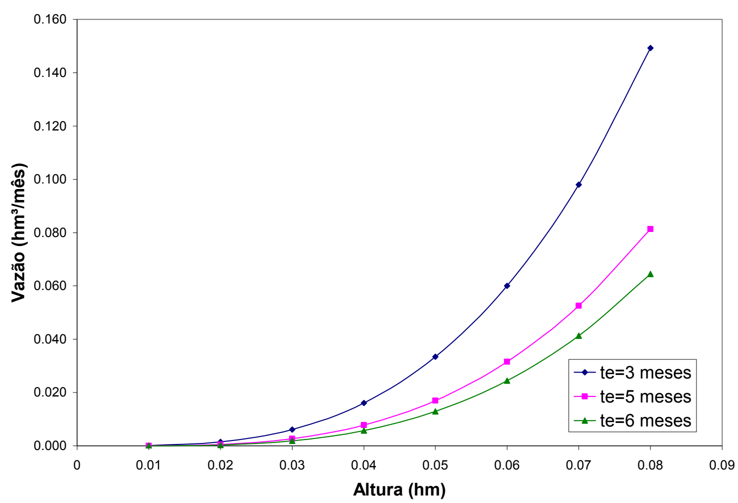

Figure 4 - Behavior of the flow with the height, for te=[3;5;6] month.

In this case, it is observed a contrary asymmetry for the two functions regarding the previous scenery, doing with that the curves that represent the largest degree of membership value moves in the direction of the membership function with upper branch, different than the last scenery.

The Figures 5 and 6 show the comparisons in the behavior of the efficiency x height and the regularized flow x height, respectively, for the time to empty with larger membership value to 4 months and 5 months.

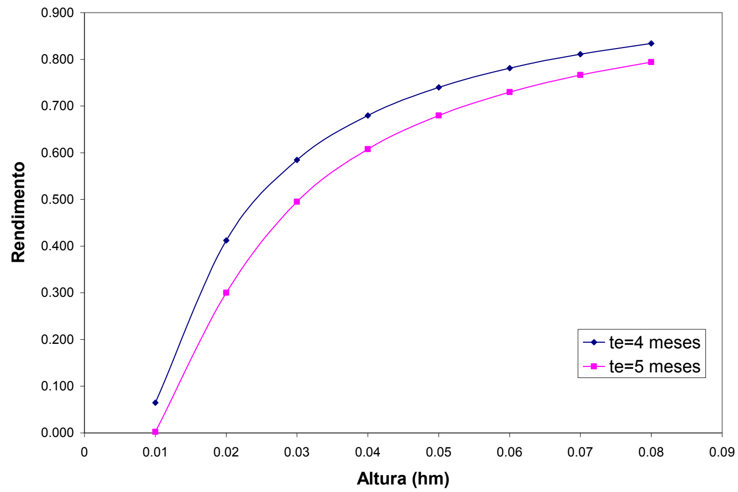

Figure 5 - Comparison among the efficiency for larger membership value.

With regard to the efficiency, the Figure display that the efficiency with larger membership value equal to 4 months is great in all extension, for any height.

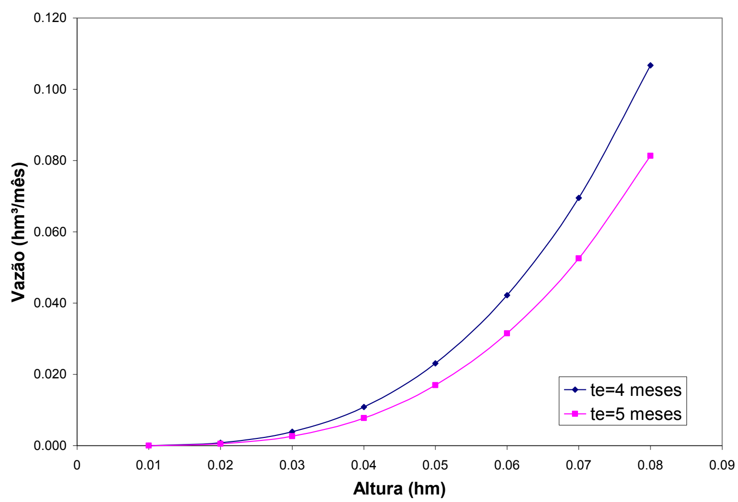

Figure 6 - Comparison among the flows of larger membership value.

On the other hand, Figure 6 shows that the behavior of the regularized flow is inverse to the behavior of the efficiency. In this case, the curve that represent te= 5 month has low value in all extension for all height considered. However, this difference increases for the largest depths. This shows that for small depths the two sceneries produce very close results.

4 Conclusions

After the analysis of the results and considering the different proposed sceneries, the following conclusions could be obtained:

When, just, the time of emptying is a number fuzzy, it was verified that the width of the base of the membership functions of the flow is larger for the largest depths. That result shows that in that scenery the risk has a softer variability than for small depths. This result shows that as bigger goes to the base of the numbers fuzzy, softer will be the behavior of the risk. That result is not confirmed for the efficiency.

In the comparison among the membership functions adopted for the time of emptying, the results showed that, for larger times of emptying with larger membership values, the regularized flow of larger membership values decreases. The results allow concluding that, as larger the depth is, bigger is this variation. With respect to the efficiency, this behavior is not defined, in other words, for larger time of emptying, with larger membership value, the efficiency, with larger membership value, happens in a close depth of 5 meters.

Finally it is possible to conclude that, according to the fuzzy methodology, for the calculation of the risk fuzzy, the time of emptying plays an important game in that evaluation. In other words, it can be said that the risk of a reservoir comes to fail is sensitive to the time of emptying. This result is in harmony with field observations, showing, like this, the capacity of the methodology fuzzy in this evaluation.

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References

Campos, J.N.B.; Studart, T., Hidrologia de Reservatórios: A Construção de Uma Teoria, Fortaleza, ASTEF, 285 p., 2006, ISBN 8575631810.

Chagas, P.F., Perspectivas da Aplicação da Teoria Fuzzy para Cálculo de Risco em Sistemas Hidrodinâmicos, Tese de Doutorado em Engenharia Civil, Universidade Federal do Ceará, Fortaleza, 2005.

Ganoulis, J.G., Engineering Risk Analysis of Water Pollution: Probabilities and Fuzzy, Set., VCH Publishers Inc., Weinheim; New York; Basel; Cambridge; Tokyo, 1994.

Lima, O. S. J., Farias, P. P.M., Belchior, A. D., 2001, Modelo Fuzzy para Análise de pontos por Função, CITS, Curitiba, Brasil.

Lima, O. S. J., Análise de Pontos por Função Fuzzy, Dissertação de Mestrado em Engenharia de Software, Universidade de Fortaleza, Fortaleza, 2002.

Oliveira Jr., H. A., Lógica Difusa: Aspectos Práticos e Aplicações, Interciência, 1999.

Suassuna, J.A., Pequena e Média Açudagem no Semi-Árido Nordestino: Uso da Água na Produção de Alimentos. Disponível em: <http://www.fundaj.gov.br/docs/text/textrop.html > Acesso em 10/08/2007.

Saavedra, O. R., Introdução aos Conjuntos Difusos – Notas de aula – Inteligência Artificial, Universidade Federal do Maranhão, 2003.

Ueta, V.; Cysne, A.P.; Campos, J.N.B., Eficiência de Acumulação de Reservatórios Anuais: Aplicação da Equação de Esvaziamento, In: XVII Simpósio Brasileiro de Recursos Hídricos, São Paulo, Anais..., 2007.

Zadeh, L. A., Fuzzy Sets – Information and Control, vol. 8 (338-353), 1965.