On grain roughness in rivers and streams

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Abstract: An effective approach to estimate the Manning's roughness in a river is to select a grain roughness value of Manning's *n* for the bed material and then add other components of roughness to the grain roughness. The grain roughness is represented by the size and shape of the grains of the material that form the stream bed. Cross section irregularities, channel alignment, vegetation, obstructions, and other factors that increase roughness and added to the grain roughness. The objective of this paper is to improve on ability to calculate grain roughness in gravel-bed rivers where the grain roughness is represented by Manning's roughness. Equations developed by G.A. Griffiths are considered to be equations that can be used to estimate the grain roughness. A method of estimating grain roughness using quantile regression shows comparable results to the Griffiths equations based on the Griffiths data and another set of data from Northern California. Data from Arizona and New York was also used to develop equations for grain roughness. These equations are very different from the Griffiths equations. A discussion of a possible reason for difference in the different equations for grain roughness is presented.

1. Introduction

The objective of this paper is to investigate equations for calculation of grain roughness in gravel and cobble-bed rivers where the grain roughness is represented by Manning's roughness. The total roughness is represented by grain roughness plus 'form roughness' with form roughness being broadly defined and includes vegetation roughness. Manning's equation is:

$$V = \frac{k_u}{n} R^{1/6} \sqrt{RS}$$

where V is the velocity, R is the hydraulic radius, S is the energy slope, n is Manning's roughness and k_u is a constant (1,.0 for metric units, and 1.4859 for traditional English units). The actual Manning's roughness for a river is larger than the roughness of the particles (grains) on the stream bed because of surface irregularities, variations in shape and size of the channel cross section, obstructions, vegetation, flow conditions, and meandering of the channel.

One approach used to estimate the Manning's roughness is to select a grain roughness value of Manning's roughness for the bed material. The grain roughness is represented by the size and shape of the grains of the material that form the stream bed (Chow, 1959). The following equation, reported by Chow to have been introduced by Cowan (1956), can be used to compute the total Manning's roughness for a channel:

$$n = \left(n_g + n_1 + n_2 + n_3 + n_4\right) m$$

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where n_g is the grain roughness, n_1 is roughness resulting from surface irregularities in the stream bed, n_2 is the roughness resulting from variations in shape and size of the channel, n3 is roughness from obstructions, and n_4 is roughness from vegetation, and m is correction factor for meandering or sinuosity of the channel. In some presentations of the term 'grain roughness' is replaced by the 'base value of the roughness' which is the valued of Manning'sroughness for a straight uniform channel. For a description of how to calculate the Manning's roughness using the Cowan equation see either Arcement and Schneider (1989) or Chow (1959). For channels with vegetation, see Phillips and Ingersoll (1998).

A second approach to calculating the total in-bank Manning roughness is characterized as consisting of two additive components: a local bed-grain roughness, n_g that varies from section to section, and a reach-wide form roughness, n_f, that vanishes monotonically at all points as the flow depth increases (Parker and Peterson ,1980). The equation is: $n = n_g + n_f$ where n is the Manning's roughness at a cross section, n_g is the grain roughness at a cross section and n_f is the form roughness for the reach.

There are numerous tables with the Manning's roughness for different types of bed material. Arcement and Schneider (1989) have a table of base values of Manning's roughness for a straight, uniform channels and, smooth channels in natural materials. Elements of that table are presented in Table 1. The original data is from Benson and Dalrymple (1967) and Chow (1959).

Table 1. Base Manning's roughness values for **s**table channels and flood plains. Original data from Benson and Dalrymple (straight uniform channels, 1967) and Chow (smooth channels, 1959).

Bed material type	**Bed material size, mm**	**Manning's roughness**		
		Straight uniform channel	Smooth channel	Strickler
Firm soil		0.025-0.032	0.02	
Coarse sand	1-2	0.026-0.035		0.013-0.015
Fine gravel			0.024	
Gravel	2-64	0.028-0.035		0.014-0.026
Coarse gravel			0.026	
Cobble	64-256	0.030-0.050		0.026-0.033
Boulder	>256	0.040-0.070		> 0.033

The Strickler equation is sometimes used to determine the Manning's roughness. The Strickler equation is $n = 0.0132(D_{50})^{1/6}$ where D_{50} is the median size of the bed material in millimeters. The Strickler equation was developed for gravel-bed rivers. The values of the roughness calculated using the Strickler equation are also shown in Table 1. It is not clear if the Strickler equation is for total roughness or grain roughness. In this paper the Strickler equation is considered to give an estimate of the grain roughness.

The Chẻzy equation is: $v = C\sqrt{RS}$ where C is the Chẻzy coefficient, R is the hydraulic radius, and S is energy slope. For Manning's equation the Chẻzy coefficient is:

$$C = \frac{k_u}{n} R^{1/6}$$

Another relation for the Chèzy coefficient uses the Darcy-Weisbach roughness, f. This relation is:

$$C = \sqrt{\frac{8g}{f}}$$

where g is the acceleration of gravity. For derivation and discussion of the Darcy-Weisbach relation, and the discussion below, see Henderson, 1966. In this paper $1/\sqrt{f}$ is called the Darcy-Weisbach factor. The Manning's roughness can be calculated from the Darcy-Weisbach factor using the equation:

$$n = \frac{k_u R^{1/6}}{\sqrt{8g}} \frac{1}{1/\sqrt{f}}$$

The unknown in the equation is $1/\sqrt{f}$. The equation for $1/\sqrt{f}$ is based on the original experiments of Nikuradse and is:

$$\frac{1}{\sqrt{f}} = \log \left(\frac{12R}{k_s}\right)^2$$

where k_s is the height of surface roughness projections and the log is to the base 10. The empirical form of the equation used in this paper is:

$$\frac{1}{\sqrt{f}} = \log\ \alpha \left(\frac{R}{D_{50}}\right)^{ß}$$

where D_{50} is the median size of the bed surface material, and alpha (α) and beta (ß) are empirical coefficients.

1.1 Regression analysis

The analysis in this paper uses least absolute deviation (LAD) regression and quantile regression. The LAD regression is done using the Simplex method of linear programming with an objective function:

$$min \sum\nolimits_{i=1,m} abs[(1/\sqrt{f}) - (1/\sqrt{f})']$$

where $\left(1\sqrt{f}\right)$ is the measured Darcy-Weisbach factor and $\left(1\sqrt{f}\right)'$ is the estimated value for the known $R/D_{50.}$ The summation is over the m set if measurements in the data set. For quantile regression the objective is the minimum sum when x percent of the measurements are below the regression line where x is the quantile.

The analysis was made using a numerical technique developed by Barrodale and Roberts (1974). The technique is Algorithm 478 from the ACM collection of algorithms. The subroutine used was written by Jon Richards of the Fort Collins Science Center (USGS).

1.2 Objective of paper

In an unpublished paper in preparation by Alonso et al (1998) the Manning's grain roughness coefficient is determined using the equation developed by Griffiths (1981) for gravel-bed rivers: This equation is:

$$n_g = \frac{k_u R^{1/6}}{\sqrt{8g}} \left[1.98 \log \left(\frac{2.42R}{D_{50}} \right) \right]^{-1}$$

where R is the hydraulic radius and D_{50} is the median size of bed surface material. If both R and D_{50} are in meters, k_u is 1.0; if in feet k_u is 1.486. The Griffiths equation yields average estimates of grain roughness for channels with pools and riffles, and is appropriate for streams where (1) the depth is large compared with the bed material size such that $5 < R/D_{50} < 200$, and (2) the stream is not transporting sediment in large quantities. The coefficients in Griffiths' formula were empirically determined from a large data base collected from New Zealand gravel-bed rivers. Griffiths mentions several special studies that can be consulted to evaluate roughness for $R/D_{50} < 5$. Some of the data sets in Griffiths´ database include form as well as grain roughness but Alonso et al regard the equation adequate for the present until a more thorough screening of the data is conducted. The objective of this paper is to investigate the effectiveness of the Griffiths equation as a predictor of grain roughness. The Darcy-Weisbach roughness factor in the Griffiths equation is $1/\sqrt{f =} \log\left(5.75(R/D_{50})^{1.98}\right)$. It is the Griffiths form of the Darcy-Weisbach roughness factor that will be investigated in this paper.

Limerinos, 1970, developed an equation for the determination of the Manning's coefficient using measured bed roughness in natural channels in Northern California. The Limerinos equation was developed as a tool to estimate the Manning's coefficient. The Darcy-Weisbach roughness factor from the Limerinos equation is $1/\sqrt{f =} \log\left(2.24(R/D_{50})^{2.0}\right)$. In this paper 80% quantile regression results will be compared to the Griffiths form of the Darcy-Weisbach roughness factor equation and the LAD results to the Limerinos form of the Darcy-Weisbach roughness factor equation.

2. Analysis of Leopold and Wolman Data

This section uses data from Leopold and Wolman (1957) to show the techniques used to develop relations that can be used to determine the grain roughness. The data used is found in Appendix H of the 1957 paper. Appendix H contains information for over 150 river reaches; of these, only 28 have the information needed to calculate Manning's roughness. The data in the appendix is for four groups of streams: 1) Wyoming streams, 2) New Mexico arroyos, 3) Brandywine Creek, Pennsylvania, and 4) Virginia and Maryland streams. The range in mean annual discharge is 4.2 to 6331 cfs. The data from the table used herein is for the depth and velocity at the mean annual discharge, the river slope, and the median size of the bed material measured using the Wolman method (Wolman, 1954).

The relation between the Manning's roughness and the median size of the bed material measured using the Leopold and Wolman method is give as Figure 1. The smallest size of bed-material that can be considered gravel is 4 mm. There four relations shown on the figure. One is the least absolute deviation relation (LAD) that gives the expected Manning's roughness for a specific median sized of the bed-material. That relation is $n = 0.0087(D_{50})^{0.50}$ where D_{50} is the median size of the bed-material in mm. The r-squared for the n versus D_{50} is 0.43. The Strickler equation is also shown on the figure. The Strickler equation is an empirical relation developed from gravel-bed river data (see

Henderson, 1967); the equation is $n = 0.0131(D_{50})^{(1/6)}$ where D_{50} is the median size of the bed material in mm. The Strickler equation can be considered a possible equation to use to estimate the Manning's grain roughness for a specific D_{50}. A second possible equation to use to estimate the grain roughness is the 20% quantile: $n = 0.0077(D_{50})^{0.43}$. The Manning's roughness for the sand sizes from the Leopold and Wolman data are also shown on Figure 1. For these five data points the roughness increases with a decrease in median size of the bed material $n = 0.0217(D_{50})^{-0.127}$. The Manning's roughness used in the equation was determined using measurements at the mean annual discharge and should be considered as a rough guide to the Manning's roughness. For a discussion of the Manning's roughness in sand-bed streams one could start with Simons and Richardson (1966).

The relation between the Darcy-Weisbach friction factor $(1/\sqrt{f})$ and the ratio of the depth to the median size of the bed material (Depth/D_{50}) for the Leopold and Wolman (1957) data is shown in Figure 2. The 80% quantile was fit to the data using quantile regression techniques. The LAD relation was fit to the data using least absolute deviation regression. The Griffiths equation is used as an equation to calculate the grain roughness and compares well to 80% quantile equation used herein as the estimator for grain resistance. Limerinos used a graphical fit of the data he collected to determine an equation to best estimate the roughness in a river, the LAD relation does the same thing. The two relations are reasonably close.

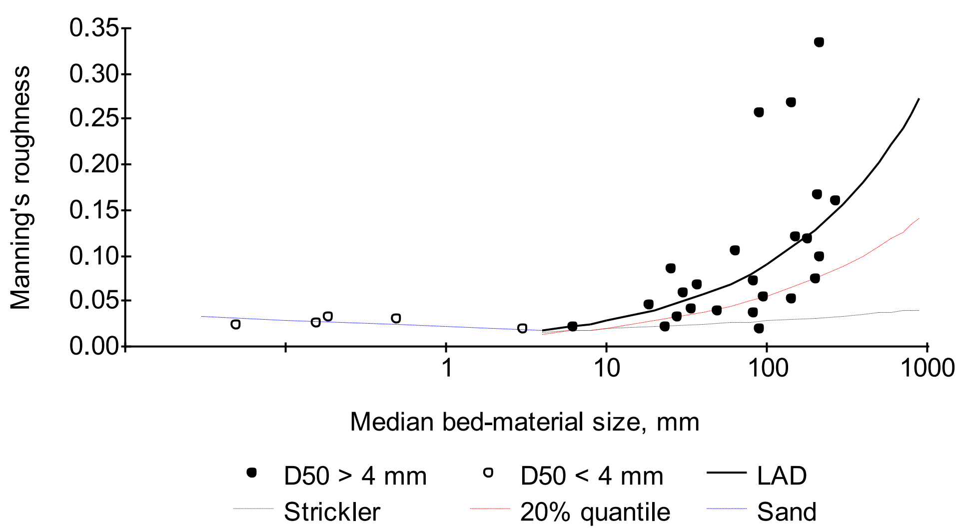

Figure 1. The relation between the Manning's roughness, n, and the median size of the bed material. Data from Leopold and Wolman (1957).

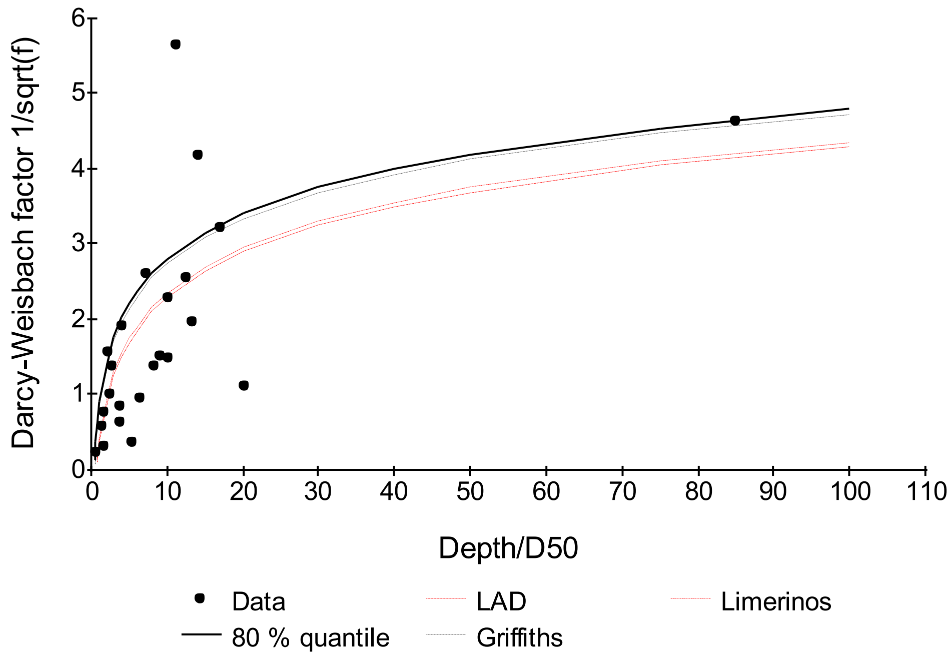

Figure 2. The relation between the Darcy-Weisbach friction factor $(1/\sqrt{f})$ and the ratio of the depth to the median size of the bed material. Data from Leopold and Wolman (1957).

3. Four Additional Data Sets

The Darcy-Weisbach grain roughness factor relations determined from analysis of four sets of data are presented in this section.

3.1 Limerinos and Griffiths data sets

As reported previously two sets of data are well known, these are the Limerinos data set collected in Northern and Central California (Limerinos, 1970) and the Griffiths collected in New Zealand (Griffiths, 1981). The relation between the Darcy-Weisbach friction factor $(1/\sqrt{f})$ and the ratio of the hydraulic to the median size of the bed material (Hydraulic radius/D_{50}) for the Limerinos data set is shown in Figure 3 and for the Griffiths data set in Figure 4. The Griffiths relations presented in this paper are based on the rigid-bed sub-set of the data as is the relation used for roughness by most user of the Griffiths paper. Both Limerinos and Griffiths fit expected value relations to their data. Both had selected reaches of rivers that were considered to be free of roughness elements other than grain roughness. Limerinos used a graphical fit of the data. The Griffiths paper does not clearly indicate how the data was fit to the equations.

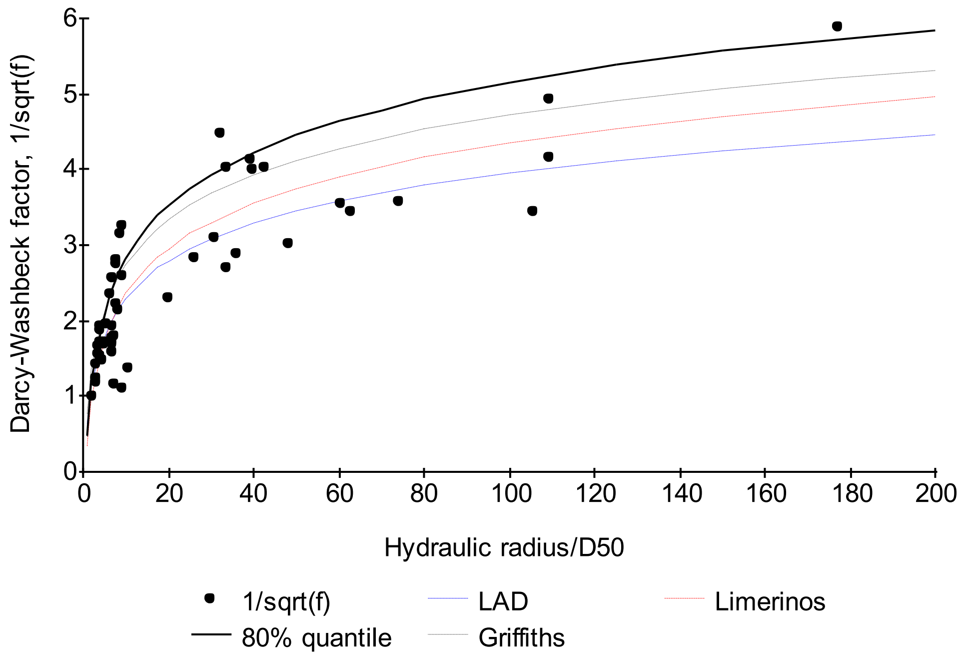

Figure 3. The relation between the Darcy-Weisbach friction factor ($1/\sqrt{f}$) and the ratio of the hydraulic radius to the median size of the bed material for data from Limerinos (1970) The Griffiths relation is from Griffiths (1981) and the Limerinos relation from Limerinos (1970).

In Figure 3 the LAD relation is an analytical fit to the data and should be close to the does the same thing as the graphical fit of Limerinos but it is not. If the beta parameter is fixed at 2.0 the alpha parameter is similar (2.24 compared to 2.40 for a LAD fit). The Limerinos equation may have been fit to the data assuming a beta value of 2. The Griffiths equation is used as an equation to calculate the grain roughness and compares reasonably well to the 80% quantile equation used herein as the estimator for grain resistance. At a R/D_{50} ratio of 100 the value of $1/\sqrt{f}$) based on the 80th percentile is 9% larger than the Griffiths equation. The 80th percentile is compared to the Griffiths equation in Figure 4. This equation will be used later herein.

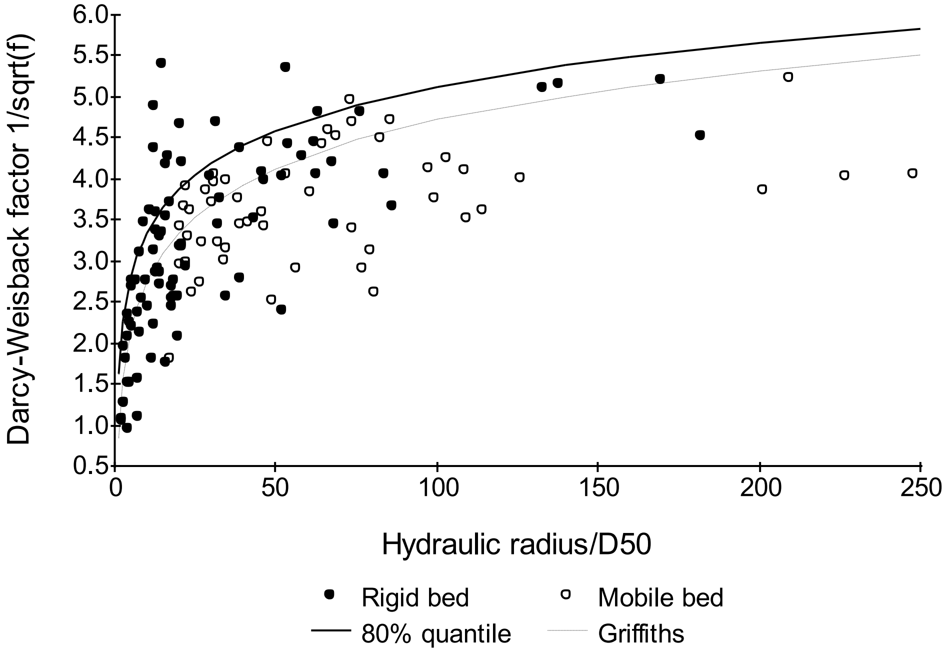

Figure 4. The relation between the Darcy-Weisbach friction factor $(1/\sqrt{f})$ and the ratio of the hydraulic radius to the median size of the bed material. The data and the Griffiths relation are from Griffiths (1981).

3.2 Arizona Data Set

The Arizona District of the U.S. Geological Survey has studied the hydraulic effects associated with channel-roughness elements in streams in Arizona (Phillips and Ingersoll, 1998). Physical and hydraulic characteristics were measured for 14 river and canal reaches in Arizona for which 37 roughness coefficients were determined. The Manning's roughness coefficients were computed from discharges, channel geometry, and water surface profiles measured at each of the sites. The data set includes constructed channels and channels with sand and fine grained substrate. Only data for the gravel and cobble rivers are used in this paper.

The purpose of the Phillips and Ingersoll equation for Manning's roughness is to relation is to calculate the 'base value' of the roughness. The Phillips and Ingersoll relation on Figure 5 is the Darcy-Weisbach factor part of their roughness equation. The 'base value' equation from Phillips and Ingersoll is functionally the same idea as grain roughness calculated using the 80th percentile.

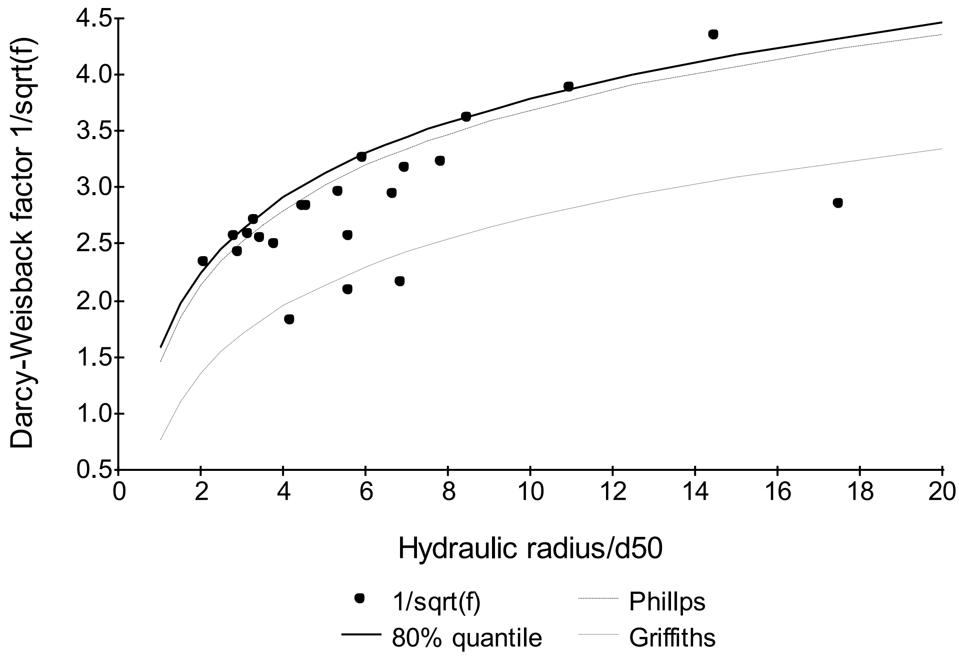

Figure 5. The relation between the Darcy-Weisbach friction factor ($1/\sqrt{f}$) and the ratio of the hydraulic radius to the median size of the bed material based on the gravel and cobble-bed river data from Phillips and Ingersoll (1998). The Phillips relation is from Phillips and Ingersoll (1998); the Griffiths relation is from Griffiths (1981).

3.3 New York Data Set

The New York data set was collected by Coon (1998) for the purpose of demonstrating how to estimate the Manning's roughness coefficients for natural stream channels with vegetated banks. The Coon report (1) summarizes related roughness coefficient studies and discusses methods commonly used for estimating Manning's roughness coefficient; (2) presents the methods of *n*-value calculation, site selection, and data collection and computation for the 21 selected sites; (3) presents photographs and computed roughness coefficients and corresponding hydraulic data for a range of discharges at each of the study sites; (4) describes the change in roughness coefficient associated with some of the major factors that influence roughness coefficients flow depth, energy gradient, size of bed material, and bank vegetation; (5) evaluates published n-value equations and their ability to reproduce the *n* values calculated from the study-site data; and (6) presents a procedure for assigning *n* values to natural channels not studied. The report does not derive equations than can be used to estimate roughness coefficients from the data collected.

The New York data has been combined with the Arizona data and the 80th percentile relation determined. The results are presented in Figure 6.

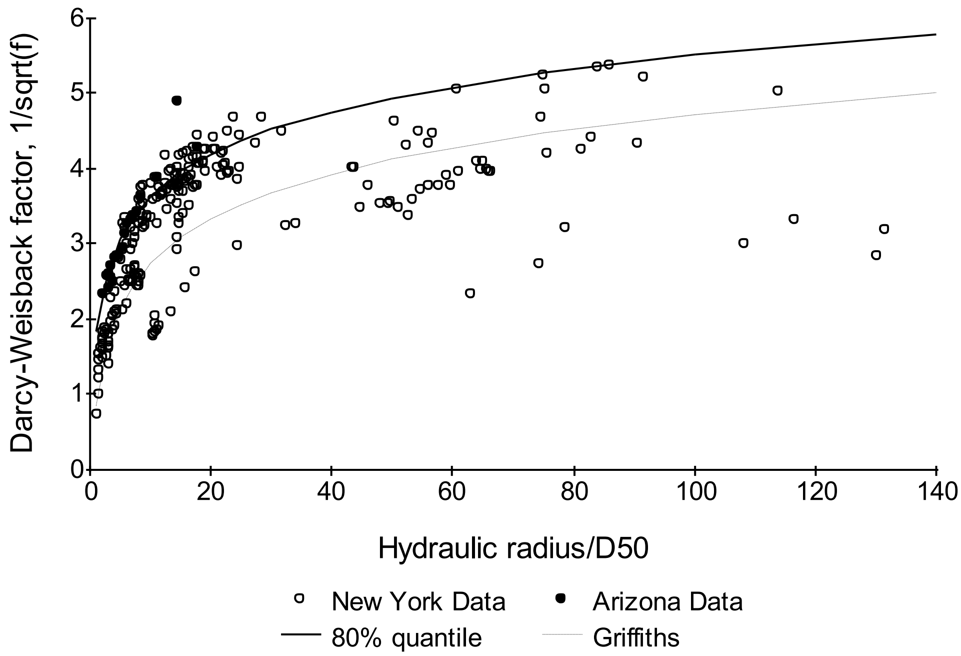

Figure 6. The relation between the Darcy-Weisbach friction factor ($1/\sqrt{f}$) and the ratio of the hydraulic radius to the median size of the bed material based on data from New York (Coon, 1998) and from Arizona (Phillips and Ingersoll, 1998).

4. Discussion and Summary

The empirical parameters in the equations for the Darcy-Washbach factor as related to the Hydraulic radius/D_{50} are summarizes in Table 2. The three grain roughness equations are from the literature. Both Griffiths and Limerinos consider their equations to be equations for grain roughness because of the selection of river reaches to study. The Phillips and Ingersoll equation is for 'base roughness' which is the same as the grain roughness.

The critical assumption made in using quantile analysis in the development of an equation for grain roughness is that a data set of roughness values as related to the ratio of the hydraulic radius to median size of bed material (R/D_{50}) will contain some roughness values that represent the grain roughness. In contrast, both Griffiths and Limerinos assumed that the only roughness in the reaches of rivers studied is grain roughness, or base roughness. In this paper the roughness relation determined using the 80% quantile analysis is the relation for grain roughness. The similarity between the 80 percentile and the Phillips and Ingersoll relation suggests the approach is effective because the objective of Phillips and Ingersoll was to develop an equation to calculate the base roughness.

The principle objective of this paper was to investigate how well the Griffiths equation for Manning's roughness represents the grain roughness. The Manning's roughness calculated using the New York+Arizona data is compared to the roughness calculated using the Griffiths equation in Figure 7 for a fixed hydraulic radius of 1.5 meters. At a hydraulic radius of 1.5 meters the New York+Arizona equation gives a Manning's grain roughness of between 11 and 23% less than the Griffiths values over the ration of R/D_{50} shown in Figure 7.

Table 2. Summary of the empirical parameters in the equation for the Darcy-Weisbach friction factor. The equation is $1/\sqrt{f} \; = \; \log\left(\alpha(R/D_{50})^{\beta}\right)$.

Alpha, α	Beta, ß	
Grain roughness equations		Equation
5.75	1.98	Griffiths,1981
2.24	2.00	Limerinos, 1970
28.8	2.23	Phillips and Ingersoll, 1998
80 % quantile relations		Data Set
40.19	1.64	Griffiths
3.09	2.33	Limerinos
37.98	2.21	Arizona
39.16	1.97	New York
56.52	1.88	New York + Arizona

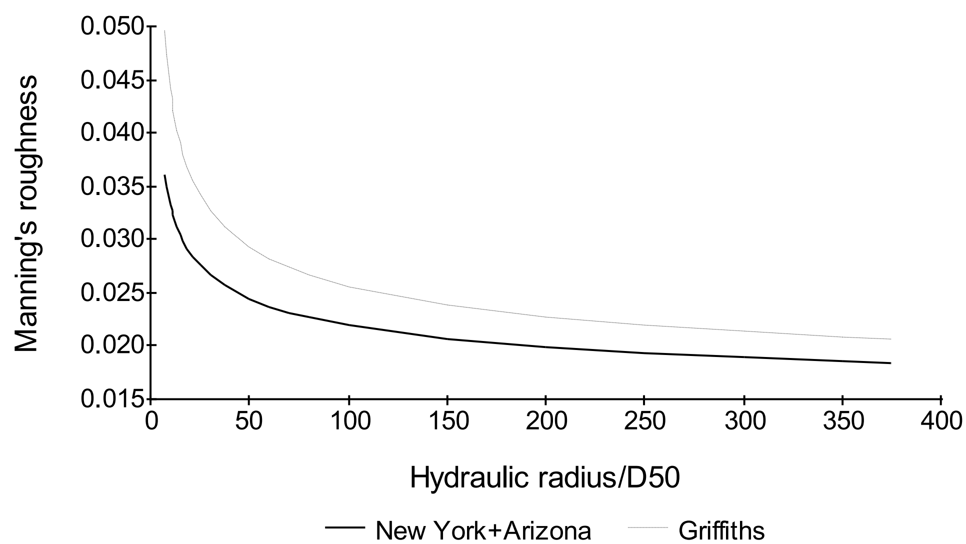

Figure 7. Relation of Manning's *n* and relative roughness (R/D_{50}) for gravel and cobble-bed stream channels in Arizona and New York compared to the roughness from the Griffiths equation for a fixed hydraulic radius of 1.5 meters.

Probably the best relation showing the grain roughness relation is the Arizona and New York data combined but there is no reason to reject results from the Griffiths equation. Phillips and Ingersoll did compare their results to the Griffiths and Limerinos equations, and another equation not described in this paper, and attempted to explain the differences. These explanations are reasonable for the Arizona data but not for the New York data. The conclusion is that the cause of the differences is not known. A possible reason for the differences might be the way the field work to determine the grain size of the bed material was done and the way the median size was determined from the field data in the office was done.

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