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On the probabilistic characterization of drought events

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Abstract. Drought characterization is an important step in water resources systems planning and management. The assessment of extreme drought events may help decision makers to set effective drought mitigation tools. Drought events can be objectively identified by three main characteristics, namely: drought duration, accumulated deficit and drought intensity. In this paper the joint cumulative distribution functions (cdf's) of accumulated deficit and duration and of intensity and duration are derived as functions of the stochastic characteristics of the underlying variable, which is assumed to be either normal, lognormal, or gamma distributed. The derived cdf's are then applied to determine the return period of critical droughts by considering jointly two drought characteristics, e.g. droughts with accumulated deficit and duration greater than or equal to some fixed values. The methodology has been tested and applied using numerical simulations and records of annual precipitation series. The results of such applications show a good correspondence between the observed and the analytical results.

1. Introduction

The probabilistic characterization of drought events is an important aspect in planning and management of water resources systems. For example, in sizing water supply storage facilities one generally considers the possible occurrence of critical droughts during the design life of the structures. Over the years, many approaches have been suggested for characterizing droughts. Yevjevich (1967) used the theory of runs to characterize droughts as a sequence of consecutive intervals where the water supply variable remains below a threshold water demand level, preceded and succeeded by values above the threshold. Thus, each drought event can be characterized by three properties, namely: drought duration, accumulated deficit, and drought intensity. Accumulated deficit, often referred to as drought magnitude, is defined as the sum of the single deficits, *i.e.* the deviations of the water supply variable from the water demand threshold, over the drought duration, whereas drought intensity is the ratio of the accumulated deficit and the drought duration.

In the analysis of multiyear droughts, the inferential approach is often unsuitable because of the limited number of drought events that can be observed from the historical records. Therefore, alternative approaches to characterize multiyear droughts involve stochastic simulations and analytical derivations of

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the probability distributions of drought characteristics and related properties such as return periods. Although the study of drought characteristics based on probabilistic approaches has been widely investigated in literature (e.g. Saldarriaga and Yevjevich, 1970; Millan and Yevjevich, 1971; Sen, 1976; Dracup et al., 1980; Cancelliere et al., 1998; Chung and Salas, 2000), the derivation of the probability distribution of accumulated deficit (or intensity) and the joint distribution of accumulated deficit and duration (or intensity and duration) are still unsolved problems, due to the mathematical difficulties that generally prevent closed form solutions. Moreover, in evaluating the return period of multiyear droughts, it is useful to consider drought events characterized by both the duration and the accumulated deficit (or duration and intensity). Also, for multiyear droughts it is not possible to identify a unique time unit (or trial) with respect to which, the exceedence probability $P[X_t>x_t]$ can be expressed, as one can usually make in flood frequency analysis, where the return period can be evaluated by the well known formula $T=1/P[X_t>x_t]$ (Fernandez and Salas, 1999).

In this regard, Fernandez and Salas (1999) provided the concept and procedure for estimating the return period of drought events with duration greater or equal to some critical value assuming a stationary two-state markov Chain. Then, Shiau and Shen (2001) assuming independent and identically distributed events, developed a procedure for deriving the return period of accumulated deficit, as the expected value of the average interarrival time between two successive events with accumulated deficit greater than or equal to a fixed value. The probability distribution of accumulated deficit conditioned on a fixed duration was assumed gamma, and the parameters were estimated through an inferential approach.

In this paper, the foregoing methodologies have been extended to the case of drought events characterized by both drought duration and accumulated deficit or drought duration and intensity. The joint distribution has been determined using the conditional distribution of drought accumulated deficit (or intensity) given duration and the marginal distribution of drought duration. The distribution of the accumulated deficit given duration has been assumed to be gamma and the parameters have been expressed as a function of the parameters of the underlying distribution of the hydrological variable (e.g. either normal, log-normal, or gamma) and the threshold water demand. Such an approach enables one to overcome the limitations of the procedure proposed by Shiau and Shen (2001), which is difficult to apply to short records. The proposed approach has been illustrated using annual precipitation records in some Italian sites.

2. Derivation of the joint probability distribution of drought characteristics

Let X_t, t=1 ,2, …, be the time series of the hydrological variable of interest and x_0 the threshold water demand level. The drought duration L_d is defined as the number of consecutive intervals where $X_t < x_0$, followed and preceded by at least one interval where $X_t \geq x_0$, whereas the accumulated deficit D_c is defined as the sum of single deficits $D_t=x_0-X_t$ over the duration L_d. It follows that the accumulated deficit can be expressed as:

$$D_c = \sum_{t=1}^{L_d} D_t = \sum_{t=1}^{L_d} \left(x_0 - X_t\right) \qquad for \qquad X_t < x_0 \tag{1}$$

The joint probability distribution of D_c and L_d can be derived from the probability density functions (pdf's) of $D_c|L_d$ and L_d as:

$$f_{D_c,L_d}\left(d_c, l_c\right) = f_{D_t|L_d=l_c}\left(d_c\right) \cdot f_{L_d}\left(l_c\right) \tag{2}$$

For stationary and independent series, the drought duration L_d is geometric distributed with parameter $p_1=1-p_0=\text{P}[X_t>x_0]$, (Llamas and Siddiqui, 1969), i.e.:

$$f_{L_d}\left(l_c\right) = p_1\left(1 - p_1\right)^{l_c - 1} \tag{3}$$

The pdf of $D_c|L_d$ could be determined if the pdf of single deficit D_t was known (Eq.1). However, such a derivation can be carried out in closed form only for simple cases. In order to overcome analytical difficulties, many authors fit a parametric distribution to empirical data of D_c or $D_c|L_d$ (e.g. Zelenhasic and Salvai, 1987; Mathier et al., 1992; Shiau and Shen, 2001).

An alternative approach consists in evaluating the parameters of the adopted distribution of D_c based on the parameters of the distribution of X_t. Indeed, the moments of $D_c|L_d$ can be expressed as functions of the moments of D_t, which in turn depend on the parameters of the variable X_t. In particular, under the assumption of serially independent X_t, the first two moments of $D_c|L_d$ are given by:

$$\text{E}\left[D_c \mid L_d\right] = \text{E}\left[\sum_{t=1}^{L_d} D_t \mid L_d = l_c\right] = l_c \ \text{E}\left[D_t\right] \tag{4}$$

$$\text{Var}\left[D_c \mid L_d\right] = \text{Var}\left[\sum_{t=1}^{L_d} D_c \mid L_d = l_c\right] = l_c \ \text{Var}\left[D_t\right] \tag{5}$$

Assuming that $D_c|L_d$ is gamma distributed, i.e.:

$$f_{D_c|L_d=l_c}\left(d_c\right) = \frac{1}{\beta\Gamma(r)}\left(\frac{d_c}{\beta}\right)^{r-1} e^{-\frac{d_c}{\beta}} \tag{6}$$

the expected value and variance of $D_c|L_d$ are respectively equal to:

$$\text{E}\left[D_c \mid L_d = l_c\right] = r \cdot \beta \tag{7}$$

$$\text{Var}\left[D_c \mid L_d = l_c\right] = r \cdot \beta^2 \tag{8}$$

Therefore, combining eqs. (4) and (5) with eqs. (7) and (8), and solving for r and β, gives:

$$r = l_c \cdot \frac{\text{E}^2\left[D_t\right]}{\text{Var}\left[D_t\right]} \tag{9}$$

$$\beta = \frac{\mathrm{Var}[D_t]}{\mathrm{E}[D_t]} \tag{10}$$

On the basis of eqs. (3) and (6), the joint probability distribution of eq. (2) becomes:

$$f_{D_c,L_d}(d_c, l_c) = \frac{1}{\beta\Gamma(r)}\left(\frac{d_c}{\beta}\right)^{r-1} e^{-\frac{d_c}{\beta}} \cdot p_1(1-p_1)^{l_c-1} \tag{11}$$

The expected value and variance of D_t can be obtained from the distribution of X_t. In general, the cumulative distribution function (cdf) of the single deficit D_t can be defined as:

$$F_{D_t}(d_t) = \mathrm{P}[(x_0 - X_t) \le d_t \mid X_t < x_0] = 1 - \frac{F_{X_t}(x_0 - d_t)}{p_0} \cdot I(d_t)_{(0,\infty)} \tag{12}$$

where $I(d_t)$=1 for $0<d_t<\infty$ and $p_0 = \mathrm{P}[X_t \le x_0]$. Taking the derivative of the cdf in eq. (12) with respect to d_t gives:

$$f_{D_t}(d_t) = \frac{1}{p_0} \cdot f_{X_t}(x_0 - d_t) \cdot I(d_t)_{(0,\infty)} \tag{13}$$

Eq. (13) shows that the pdf of single deficit is the truncated pdf of X_t. Thus the k^{th} moment is given by:

$$\mathrm{E}[D_t^k] = \frac{1}{p_0}\int_0^\infty d_t^k \cdot f_{X_t}(x_0 - d_t) \cdot \mathrm{d}\, d_t \tag{14}$$

Hence, the first two moments of D_t for virtually any distribution of X_t can be obtained from eq. (14). Substituting those moments into eqs. (9) and (10) will yield the desired parameters r and β of the distribution of accumulated deficit. Table I gives the expressions of r and β for three different distributions of X_t, namely normal, lognormal and gamma (Bonaccorso et al., 2003).

In addition, the threshold demand levels x_0 can be expressed as (Yevjevich, 1967):

$$x_0 = \mu_x - \alpha\sigma_x = \mu_x(1 - \alpha C_v) \tag{15}$$

where σ_x and C_v are respectively the standard deviation and the coefficient of variation of X_t and α is a dimensionless coefficient. Furthermore, it is known that

- for X_t ~ lognormal (μ_y, σ_y) $\Rightarrow$ $\sigma_y = \sqrt{\ln(C_v^2 + 1)}$
- for X_t ~ gamma (μ_x, σ_x) $\Rightarrow$ $r_x = 1/C_v^2$

then, the parameters r and β for each of the three distributions considered in Table I can be written in the following general form:

$$r = l_c\varphi(\alpha, C_v) \tag{16}$$

$$\beta = \mu_x\, \delta(\alpha, C_v) \tag{17}$$

where φ and δ are functions that depend on the adopted distribution. Therefore, introducing the expressions (16)-(17) and the dimensionless accumulated deficit $d_c^*=d_c/\mu_x$ instead of d_c in eq. (7), it follows that the joint distribution of the dimensionless accumulated deficit and duration is completely defined by α and C_v.

The conditional distribution of drought intensity I given L_d can also be derived (Salas et al., 2003). Indeed, since the drought intensity is the ratio of accumulated deficit to drought duration, i.e. $I=D_c/L_d$, if $D_c|L_d$ ~ gamma (r, β), it can be shown that the pdf of $I|L_d$ is given by:

$$f_{I|L_d=l_c}(i)=\frac{1}{\beta\Gamma(r)}\left(\frac{l_c i}{\beta}\right)^{r-1}e^{-\frac{l_c i}{\beta}} \tag{18}$$

which is also gamma distributed, i.e. ~ gamma (r, β/l_c). Thus, the joint pdf of intensity and duration can be found in a similar fashion as in eq. (7):

$$f_{I,L_d}(i,l_c)=\frac{1}{\beta\Gamma(r)}\left(\frac{l_c i}{\beta}\right)^{r-1}e^{-\frac{l_c i}{\beta}}\cdot p_1(1-p_1)^{l_c-1} \tag{19}$$

where the parameters r and β are the same as in the previous case.

Table I. Parameters of the gamma cdf of $D_c|L_d$ for different distributions of X_t

Distribution of X_t	**r**	**β**	**Other parameters**
Normal (μ_x, σ_x)	$\dfrac{l_c\left(-\alpha+\frac{\phi(-\alpha)}{p_0}\right)^2}{\left(\alpha\frac{\phi(-\alpha)}{p_0}-\frac{\phi^2(-\alpha)}{p_0^2}+1\right)}$	$\dfrac{\mu_x C_v\left(\alpha\frac{\phi(-\alpha)}{p_0}-\frac{\phi^2(-\alpha)}{p_0^2}+1\right)}{\left(-\alpha+\frac{\phi(-\alpha)}{p_0}\right)}$	$p_0=\Phi(-\alpha)$
Lognormal (μ_y, σ_y)	$\dfrac{l_c\left(1-\alpha C_v-\frac{\Delta}{p_0}\right)^2}{\left(-\frac{\Delta^2}{p_0^2}+e^{\sigma_y^2}\frac{\Psi}{p_0}\right)}$	$\dfrac{\mu_x\left(-\frac{\Delta^2}{p_0^2}+e^{\sigma_y^2}\frac{\Psi}{p_0}\right)}{\left(1-\alpha C_v-\frac{\Delta}{p_0}\right)}$	$p_0=\Phi\left[\frac{1}{2}\sigma_y+\frac{\ln(1-\alpha C_v)}{\sigma_y}\right]$ $\Delta=\Phi\left[-\frac{1}{2}\sigma_y+\frac{\ln(1-\alpha C_v)}{\sigma_y}\right]$ $\Psi=\Phi\left[-\frac{3}{2}\sigma_y+\frac{\ln(1-\alpha C_v)}{\sigma_y}\right]$
Gamma (r_x, β_x)	$\dfrac{l_c\left(1-\alpha C_v-\frac{\Theta}{p_0}\right)^2}{\left(-\frac{\Theta^2}{p_0^2}+\frac{\Omega}{p_0}(C_v^2+1)\right)}$	$\dfrac{\mu_x\left(-\frac{\Theta^2}{p_0^2}+\frac{\Omega}{p_0}(C_v^2+1)\right)}{\left(1-\alpha C_v-\frac{\Theta}{p_0}\right)}$	$p_0=\mathbf{P}[r_x, r_x(1-\alpha C_v)]$ $\Theta=\mathbf{P}[r_x+1, r_x(1-\alpha C_v)]$ $\Omega=\mathbf{P}[r_x+2, r_x(1-\alpha C_v)]$

3. Assessment of drought return period

The return period can be defined as the average elapsed time or mean interarrival time between occurrences of critical events (e.g. Lloyd, 1970; Loaciga and Mariño, 1991; Shiau and Shen, 2001), for instance drought events with accumulated deficit (or intensity) and duration greater than or equal to fixed values.

The interarrival time is defined as the period between the beginning of a drought and the beginning of the next one, namely the sum of the duration of drought period L_n and non drought (wet) period $L_{w.}$. The definition of the interarrival time of a critical drought is explained in figure 1.

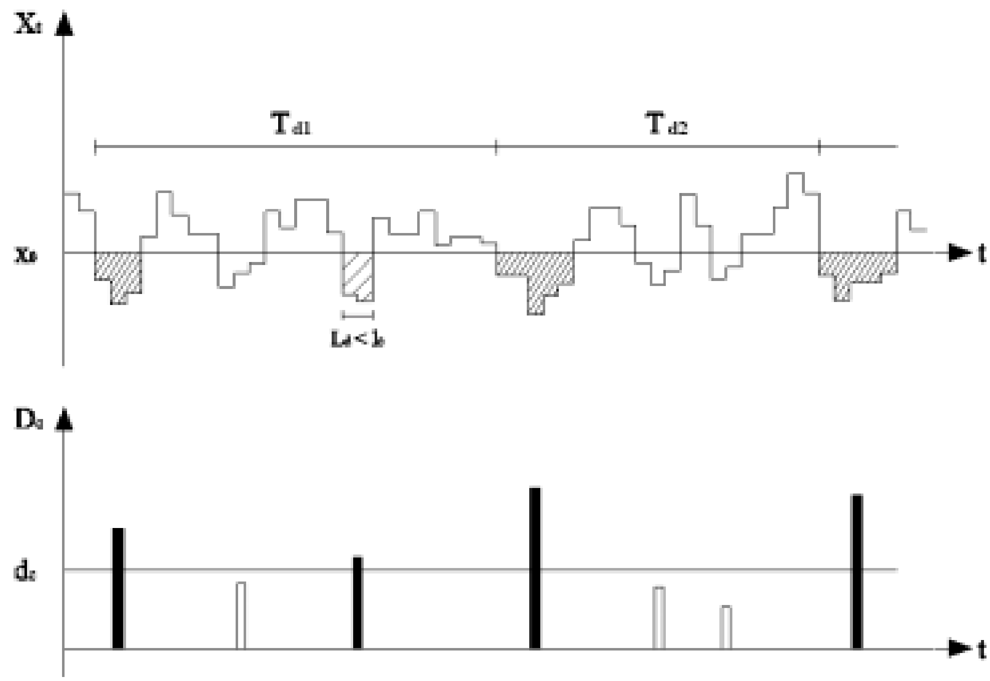

Figure 1. Interarrival time T_d between drought events with severity > d_c and duration ≥ l_c (represented by closer hatched areas)

Therefore, the interarrival time between two critical drought events can be analytically expressed as:

$$T_d = \sum_{i=1}^{N} L_i \tag{20}$$

where L_i is the interarrival time between any two droughts (droughts of any severity) and N is the number of droughts preceding the next critical drought. According to the above definition, the return period of the critical event can be computed as the expected value of T_d, i.e.

$$\mathrm{E}[T_d] = \mathrm{E}\left[\sum_{i=1}^{N} L_i\right] = \mathrm{E}[N]\,\mathrm{E}[L] \tag{21}$$

The durations of periods of droughts and non-droughts can be modelled by a geometric distribution with parameters p_1 and p_0, respectively. It follows:

$$\mathrm{E}[L] = \mathrm{E}[L_d] + \mathrm{E}[L_w] = \frac{1}{p_1 p_0} \tag{22}$$

With reference to a critical drought A, it can be shown that N is also geometric with probability mass function given by

$$P(N = n) = P[A] \cdot (1 - P[A])^{n-1} \tag{23}$$

where $P[A]$ is the occurrence probability of A. Hence eq. (21) can be rewritten as:

$$E[T_d] = \frac{1}{p_1 p_0} \cdot \frac{1}{P[A]} \tag{24}$$

In particular, with reference to four types of critical drought events, the following expressions can be found (Salas et al., 2003):

1) for drought event $A = \{D>d_c \text{ and } L_d = l_c \ (l_c=1,2,\ldots)\}$:

$$P[D_c > d_c, L_d = l_c] = \int_{d_c}^{\infty} f_{D_c,L_d}(z, l_c)\,\mathrm{d}z = \left[1 - \mathbf{G}\left(l_c\varphi, \frac{d_c^*}{\delta}\right)\right] \cdot p_1 (1-p_1)^{l_c-1} \tag{25}$$

where $\mathbf{G}(\cdot)$ is the incomplete gamma function (*Abramowitz & Stegun*,1965)

2) for drought event $A = \{D>d_c \text{ and } L_d \geq l_c \ (l_c=1,2,\ldots)\}$:

$$P[D_c > d_c, L_d \geq l_c] = \int_{d_c}^{\infty} \sum_{l=l_c}^{\infty} f_{D_c,L_d}(z, l)\,\mathrm{d}z = \sum_{l=l_c}^{\infty} \left[1 - \mathbf{G}\left(l\varphi, \frac{d_c^*}{\delta}\right)\right] \cdot p_1 (1-p_1)^{l-1} \tag{26}$$

3) for drought event $A = \{I > i \text{ and } L_d = l_c \ (l_c=1,2,\ldots)\}$:

$$P[I > i, L_d = l_c] = \int_{i}^{\infty} f_{I,L}(z, l_c)\,\mathrm{d}z = \left[1 - \mathbf{G}\left(l_c\varphi, \frac{l_c i^*}{\delta}\right)\right] \cdot p_1 (1-p_1)^{l_c-1} \tag{27}$$

4) for drought event $A = \{I > i \text{ and } L_d \geq l_c \ (l_c=1,2,\ldots)\}$:

$$P[I > i, L_d \geq l_c] = \int_{i}^{\infty} \sum_{l=l_c}^{\infty} f_{I,L_d}(z, l)\,\mathrm{d}z = \sum_{l=l_c}^{\infty} \left[1 - \mathbf{G}\left(l\varphi, \frac{l_c i^*}{\delta}\right)\right] \cdot p_1 (1-p_1)^{l-1} \tag{28}$$

where, for the sake of simplicity, the parameters of the gamma distributions have been indicated as $r=l_c\varphi$ and $\beta=\mu_x\delta$, whereas $i^*=i/\mu_x$ is the dimensionless intensity. Therefore from equations (24)-(28) the return period of various drought events can be found. It follows that the return period depends only on the threshold coefficient α and the coefficient of variation C_v of the underlying hydrological series. It should be noted that despite the apparent complexity of the above expressions, the integrations can be carried out efficiently making use of numerical tools for the gamma cdf that are available in most mathematical and statistical software.

The procedure describe above has been applied using historical series of precipitation to assess the return periods of different types of drought events. In particular, a threshold demand equal to the long term mean of X_t (i.e. α=0) has been considered for drought identification

4. Application

The proposed procedure has been applied using annual precipitation records from three stations in Italy, namely Petralia, Milano Brera, and Agrigento,. Table II shows some sample statistics for the referred data. The application of the Chi- square test suggested that the historical precipitation series may be modeled by either the normal, log-normal, or gamma distributions. Also the annual precipitation series was tested to be serially uncorrelated.

Table II. Sample statistics of the annual precipitation series used in the study

Station	Period of record [years]	Mean [mm]	Coefficient of variation C_v
Petralia	116	775.0	0.24
Milano Brera	234	997.6	0.20
Agrigento	111	498.0	0.27

Then, three 50,000 years of synthetic precipitation records were generated from the referred distributions. The return periods of droughts obtained from the historical and generated records (estimated by averaging the interarrival times between critical droughts) and from the proposed equations were compared. A threshold level x_0 equal to the long term mean (i.e., α=0) has been considered for drought identification.

Figures 2, 3, and 4 show, for stations Petralia, Milano Brera, and Agrigento, the return periods of droughts specified by Eqs.(25)-(28) and identified in Figs. (2)-(4) as I) $A = \{D>d_c$ and $L_d = l_c\}$, II) $A = \{D>d_c$ and $L_d \geq l_c\}$, III) $A = \{I > i$ and $L_d = l_c\}$ and IV) $A = \{I > i$ and $L_d \geq l_c\}$), respectively. The figures show the results obtained for various values of d_c^* and i^* except the results for the historical series are available only for d_c^*=0. In general, a good correspondence between the results obtained from the historical records and those determined from the generated samples and from eqs. (25)-(28) are evident.

From figures 2.I, 3.I, and 4.I, one may observe that for a given drought duration l_c, the return period $T \rightarrow \infty$ as d_c^* increases, which means that for estimating the return period T for large values of d_c^* a very long sample may be required. Indeed, it can be noted that the difference between the results obtained from data generation (dashed lines) and those obtained analytically (continuous lines) increases significantly with d_c^* and is more relevant for short drought duration, due to the fact that not many drought episodes are identified in the series.

Figures 2.II, 3.II and 4.II show that as l_c increases all return period curves apparently converge to a single curve that is independent of d_c^*. Figures 2.III, 3.III and 4.III and 2.IV, 3.IV and 4.IV show that the return period curves are increasing function of l_c and i^*. Analytical and generated results show a good correspondence for all values of i^* for both the Petralia and Agrigento stations. In the case of the Milano Brera station, there is a noticeable difference between the results obtained for the case $i^* \geq 0.30$. It is worth noting that, except perhaps for the case of Milano Brera, the values of T estimated from the historical sample for $d_c^* = 0$ are generally not reliable for $l_c > 2$ or 3, because of the limited number of drought episodes which can be observed from the historical sample.

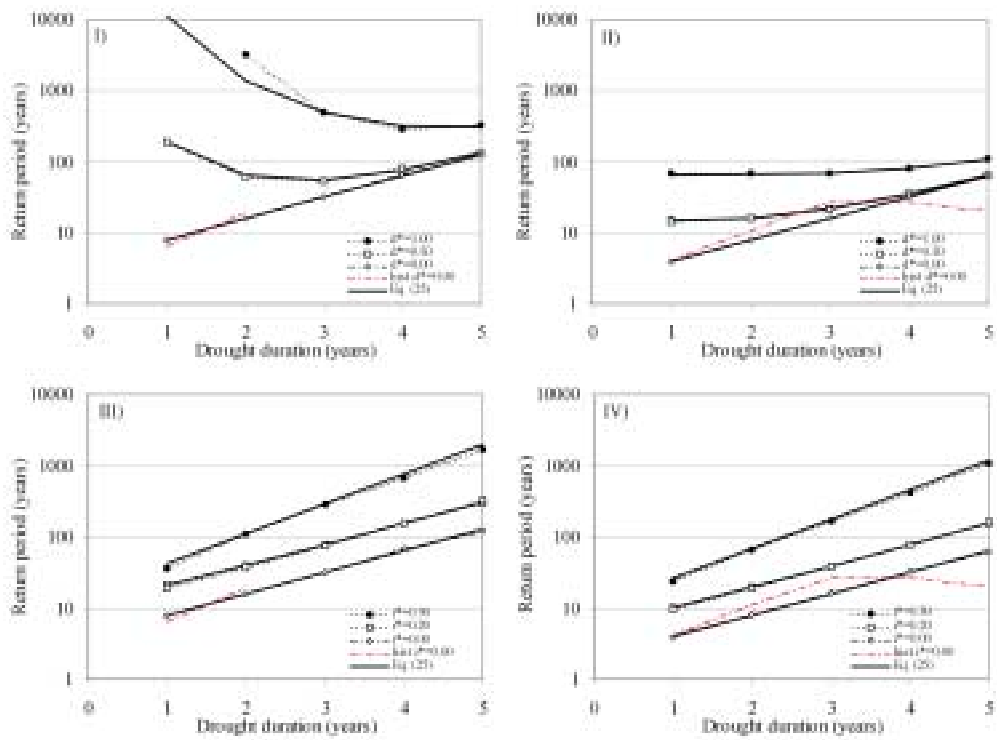

Figure 2. Return period of drought events obtained from generated annual precipitation of Petralia (normal) and from eq. (25) for various values of d_c^* and i^* , and return period from the observed historical sample for d_c^* =0 and i^*=0

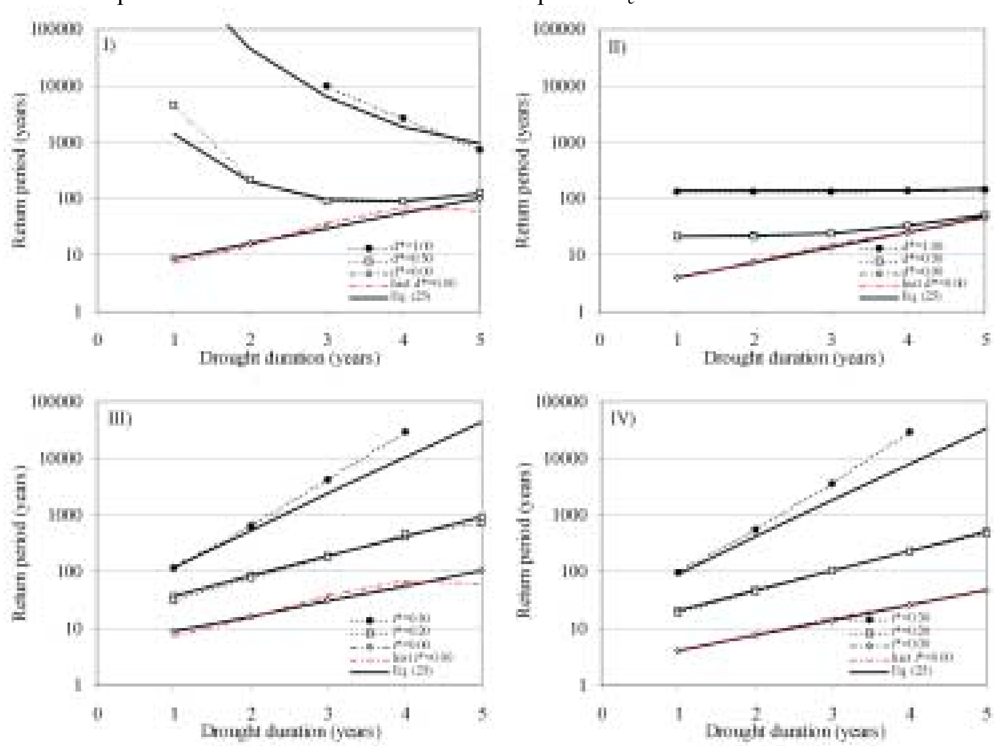

Figure 3. Return period of drought events obtained from generated annual precipitation of Milano Brera (lognormal) and from eq. (25) for various values of d_c^* and i^* , and return period from the observed historical sample for d_c^* =0 and i^*=0

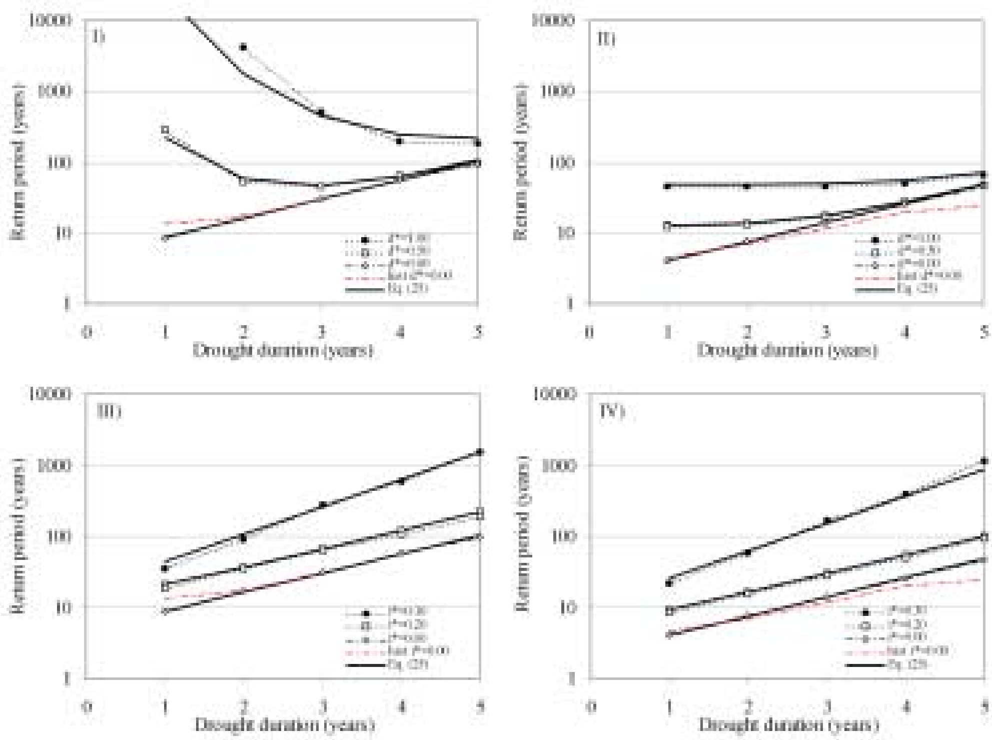

Figure 4. Return period of drought events obtained from generated annual precipitation of Agrigento (gamma) and from eq. (25) for various values of d_c^* and i^* , and return period from the observed historical sample for d_c^* =0 and i^*=0

5. Conclusions

The analysis of drought events is extremely important in water resources planning and management. In spite of the large number of studies that have been carried on the subject, the exact derivation of the probabilistic structure of drought characteristics is still an unsolved problem, especially when both duration and accumulated deficit (or intensity) are taken into account. In this paper a methodology to derive the probability distribution of drought episodes considering both drought duration and accumulated deficit (or intensity) and the ensuing return period are presented. The derivations are based on the conditional distribution of accumulated deficit (or intensity) given duration, which has been taken as gamma distributed, and the distribution of the duration, which, for independent and stationary series, is known to be geometric. The parameters of the gamma distribution have been determined as functions of the coefficient of variation of the underlying hydrological variable (considering either normal, lognormal and gamma distributed) and the threshold demand level.

The proposed methodology enables one to overcome the difficulties related to estimation based on historical records alone. In fact, even when using generated samples sometimes, for example, in cases of short drought duration and large deficits or intensities, the estimation of return periods may not be accurate. The proposed approach for modeling drought events and estimating the corresponding return periods has been tested using generated samples and using precipitation records of three stations in Italy. For the most part the results showed very good results. We are currently investigating an apparent discrepancy of the results obtained for one of the sites for values of dimensionless intensities bigger or equal to 0.3.

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