Regime equations for Mountain Streams in the Cauca Region of Colombia

Ana C. Arbeláez[1],
Civil Engineer, MSc.

María Elvira Guevara A[2].
Civil Engineer, MSc.

Lilian Posada G[3] .
Civil Engineer, MSc, Ph. D

Luís Jorge González M[2],
Agricultural Engineer

Carlos A. Gallardo B[2],
Agricultural Engineer. MSc.

Abstract. Knowledge of the dimensions of a stable stream channel during a specific period of time and of its corresponding forming discharge allows determining parameters of regime equations for a region, which enhance the design of channels in other zones with similar characteristics.
Nowadays, the state-of-the-art for regime channel does not include the typical streams of the Department of Cauca characterized by relative steep slopes, high annual precipitation in the order of 2500 – 3000 mm, high discharge and varied geology. In this article, we present regime type expressions obtained from a research done in mountain streams of the Department of Cauca, Colombia.

Key Words: Regime equations, Stable channel, Forming discharge

1 Introduction

The knowledge of the existent relationships among Hydrology and Morphology of the river basin is of fundamental importance to understand the

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[1] Área Metropolitana del Valle de Aburra. This research was developed while the author was professor at Universidad del Cauca. Colombia
Medellín, Calle 75sur No 52-101. Colombia
Tel: (574) 3093878 e-mail: acarbela@yahoo.com
[2] Professor Facultad de Ingeniería Civil, Universidad del Cauca.
Popayán Colombia Calle 5 No. 4-70 - Tel. +57 (2) 8209900
Email: mguevara@unicauca.edu.co
[3] Associate Professor Universidad Nacional de Colombia. Sede Medellín.
Medellín, Colombia. South America. Cl 44B 82-46. Tel. +57 (4) 4118213
E-mail: lposada@unalmed.edu.co

processes of channel formation, bed agradación or degradation, and channel stability for improving design of channels structures.

The Regime theory of channels has not been updated to include high gradient streams like those of the Cauca region in south western Colombia that are characterized by steep slopes, annual precipitation depths of the order of 2500 mm - 3000 mm, high flow regimes, running through a complex geology.

Selected Regime equations were considered to be applied to the study site and expressions for gravel mountain streams with mean bed material diameter in the range of 19 to 230 mm, bankfull water discharge between 7 and 58 m^3/s, and bed slope between 0.01 and 0.072 m/m were developed during the present research.

2 The Study Site

The Andes range in the state of Cauca, at the southwestern Colombia is the origin of some of the larger rivers in the country, such as Magdalena, Cauca, Patía and Caquetá (Figure 1); the altitude of the state varies from 900 to 3500 meters under sea level.

Some typical mountain streams tributaries to Cauca River in the state of Cauca with enough hydrologic and hydraulic information were sampled to obtain sediment and bankfull flow data which includes discharge, channel geometry (slope, width, depth), and characteristics of vegetation.

Figure 1 Study Site Location.

3 Regimen Theory

The form of the natural and irrigation channels have been of paramount interest in the geomorphologic studies in order to design stable channels or “regime channels”. A stable channel is that for which there is no significant change of their geometric (width, flow depth, slope and cross sectional area) and flow (velocity, discharge) variables through a long period of time. Being strict, natural channels are constantly exhibiting aggradation and degradation processes for the concept of stability is applied to time scales relatively high, from 1 to 10 years (Mejía, G, 2001).

The Regime theory has its beginnings since 1895 when 1895 when an anglo-hindu group of investigators (Martin, 1997) observed that the irrigation channels (under constant flow) acquired a stable geometry through time. Later on, the Regime theory was extended to natural channels, by using the concept of forming discharge.

That concept establish that a stream has a tendency to obtain a stable equilibrium under certain constant environmental conditions during a period of time of years and any change in the hydrologic or sediment flow regime will conduct to a measured response in the channel (erosion or deposition).

In spite of the diversity of variables controlling the phenomenon, the main expressions of the Regime theory are focused primarily to establish empiric relationships involving the channel geometric form (width, depth, slope), the forming discharge, the sediment load, the grain distribution of bed, and the resistance of the banks.

There are numerous expressions for rivers in Regime (Mejía, G., 2001) that can be applied to mountain rivers according to their particular conditions of its development. There are grouped in either bivariate or trivariate sets, according to the consideration of slope as an independent or dependent variable, respectively:

Bivariate group

$$W=\alpha_1 {Q_b}^{\beta 1} {D_{50}}^{\lambda 1} \quad H=\alpha_2 {Q_b}^{\beta 2} {D_{50}}^{\lambda 2} \quad S=\alpha_3 {Q_b}^{\beta 3} {D_{50}}^{\lambda 3} \tag{1}$$

Trivariate group

$$W=\alpha_1 {Q_b}^{\beta 1} {D_{50}}^{\lambda 1} S^{\psi 1} \qquad H=\alpha_2 {Q_b}^{\beta 2} {D_{50}}^{\lambda 2} S^{\psi 2} \tag{2}$$

Parker (2004), proposed no dimensional relationships for the Regime channel geometry based upon a wide database that includes gravel and sand bed rivers. He identified a behavior for gravel different than that for sand bed rivers, as follow: For gravel bed rivers.

$$W^*=\alpha_1 {Q_b}^{*\beta 1} \quad H^*=\alpha_2 {Q_b}^{*\beta 2} \quad S=\alpha_3 {Q_b}^{*\beta 3} \tag{3}$$

$$\text{where } W^*=\frac{W}{D_{50}} \quad H^*=\frac{H}{D_{50}} \quad {Q_b}^*=\frac{Q_b}{\sqrt{gD_{50}}\,D_{50}^2} \tag{4}$$

Parker (2004) suggested using mean values of depth measurements for stable reaches.

Table 1 shows a summary of the different coefficients included on regime expressions 1 to 4.

Table 1 Summary of the coefficients for some regime type equations

Variable	Author				
	Yalin, 1992	Bray, 1982	Kellerhals[4], 1967	Mejía, 2001	Parker[5], 2004
α_1	1.5	6.21	3.26	0.822	4.87
β_1	0.5	0.53	0.5	0.031	0.461
λ_1	-0.25	-0.07		-0.135	
ψ_1				-0.135	
α_2	0.15	0.31	0.417	0.129	0.368
β_2	0.43	0.33	0.7	0.325	0.405
λ_2	-0.07	-0.03	-0.12	0.069	
ψ_2				-0.169	
α_3	0.55	0.001	0.00015		0.0976
β_3	-0.43	-0.33	-0.4		-0.341
λ_3	1.07	0.59	0.92		

The metric system of units (m) is used for W and H; (mm) for D_{50} and (m^3/s) for discharge. Since all the equations are empirical expressions, care has to be taken to use the same system of units adopted for its authors and that system is not always explicitly announced.

4 Data Processing And Analysis Of Results

4.1 Field campaigns

Flow measurement campaigns were conducted on some stable reaches of tributary streams to Cauca River. Stable straight reaches with easily identifiable bank full indicators were selected. Discharge measurements included the field determination of hydraulic and geometric variables such as width, flow depth, wet perimeter, cross sectional area, bed topographic slope, velocity, discharge, and the grain size distribution of bed material was determined according to the Wolman pebble count technique (Wolman, 1954) which is recommended for gravel bed rivers. Table 2 summarizes the field measured variables.

During the field measurements, the forming discharge indicators were identified in most of the sites, looking at the floodplain landscape, changes in slope of the banks, changes in bed material size at the point bars, vegetation indicators, and so on. For the last one, there were certain difficulties since there was a lot of lichen and riparian vegetation that hide the proper indicators for bankfull levels. Due to the particular characteristics of tropical hydrology where a quicker growing of lichen and riparian vegetation is to be expected, the return period associated to forming discharge in tropical climates does not

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[4] Characteristic Diameter D_{50}

[5] Dimensionless variables

necessarily corresponds to those periods on temperate regions. Then, the best indicator for defining bank full level was the change on the sidewall slope.

Table 2 Summary of geometric and hydraulic variables measured during the flow measurement campaigns.

Stream Name	**Area km²**	W **(m)**	H **(m)**	A **(m²)**	V **(m/s)**	Q **m³/s**	D_{50} **mm**
Cauca	292.84	21.70	0.33	7.06	0.66	4.63	172.52
Ovejas1	261.41	13.80	0.50	6.88	1.02	7.02	43.5
Ovejas2	261.41	15.40	0.35	5.35	1.32	7.05	64.80
Piendamó	152.07	10.90	0.89	9.74	0.21	2.06	192.00
Saté	19.08	4.50	0.46	2.07	0.33	0.68	56.00
Palacé	245.01	12.75	0.56	7.16	0.50	3.59	107.79
Cofre1	217.37	11.80	0.70	8.29	0.55	4.55	104.00
Cofre2	217.37	11.00	0.40	4.44	0.79	3.51	114.67
Mondomo1	218.59	12.20	0.43	5.23	0.87	4.56	59.00
Mondomo2	218.59	12.70	0.36	4.54	1.09	4.95	44.34
Piedras1	30.64	6.50	0.34	2.20	0.30	0.66	193.28
Piedras2	30.64	4.90	0.29	1.42	0.62	0.88	230.40
Molino	25.60	11.30	0.14	1.54	0.61	0.94	19.60

4.1.1 Magnitude and Frequency of Forming Discharge.

Field information was used for estimating forming discharges by considering the Manning's term $S^{0.5}/n$ to be constant during high flow levels. S is the hydraulic gradient and n is the Manning's coefficient of roughness. Table 3 presents a summary of hydraulic and geometric variables characterizing the Forming Discharge for Cauca River tributary streams. The return period, T_R was determined by frequency analysis for the historical records of instantaneous maximum discharge (Arbeláez et al., 2005) available for the region.

When comparing the theoretical values for the return period of bank full discharge, 1-2 years, with the corresponding estimated values of T_R, a good agreement is observed. A mean value of 1.4 years for the observed return period of the study streams was found.

Table 3 Forming discharge variables for the study streams.

Stream name	W **(m)**	H **(m)**	A **(m²)**	V **(m/s)**	Q **(m³/s)**	T_R **(years)**
Cauca	29.00	1.06	30.83	1.89	58.22	2.45
Ovejas1	16.60	1.23	20.40	2.40	48.93	1.49
Ovejas2	24.60	1.08	26.62	1.87	49.69	1.52
Piendamó	13.52	1.50	20.31	1.19	24.13	1.08
Saté	6.71	1.34	9.00	0.77	6.96	1.50
Palacé	14.55	1.55	22.50	1.19	26.82	1.17
Cofre1	15.51	1.14	17.67	0.99	17.56	1.21
Cofre2	15.03	0.87	13.02	1.09	14.16	1.10
Mondomo1	14.14	0.73	10.37	2.13	22.07	1.01
Mondomo2	13.48	1.19	16.03	1.75	28.03	1.06
Piedras1	12.12	0.97	11.79	0.88	10.40	1.30
Piedras2	10.77	0.81	8.76	0.97	8.52	1.17
Molino[6]	12.45	1.24	15.42	2.53	39.03	N/A

4.2 Regime type Relationships.

The observed values of main geometric and hydraulics variables describing the forming discharge of the study streams (Table 1) are compared with the theoretical corresponding variables reported by rivers of similar characteristics (gravel bed rivers) and the results are very poor indicating that none of the theoretical Regime equations represents the behavior of the tropical streams used on present research. The regime equations proposed by Kellerhals, Yalin and Parker for the width of the channel represent quite well the tropical streams of the Cauca region in Colombia, but none of the theoretical regime equations considered on the present study for modeling the slope and the depth of the forming discharge showed a convenient result for the tropical gravel bed streams.

To determine the correct type of equations to model the forming discharge variables it is necessary to consider the statistical limitations due to the size of sample since the present investigation only have a number of 13 gravel bed streams. For a good multiple correlation analysis a large sample size is very important since the larger number of dependent variables, the greater the simple size is necessary to have statistically reliable results. For this reason, the criteria to select the theoretical models were:

a) Models considering the slope as an independent variable, for which bivariate type of bank full discharge and characteristic sediment diameter for multiple correlation analysis is expected.

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[6] Ungauged stream. No frequency analysis was done

b) Single variable models using dimensional analysis as proposed by parker (2004).

The range of data for gravel bed tributaries of Cauca River are:

Bed sediment diameter (D_{50}) between 19 mm and 230 mm
Bank full discharge (Q_b) between 7 and 58 m³/s
Bed slope (S) ranging from 0.01 to 0.072 m/m

The metric system of units is used on the proposed expressions where width (W) and Depth (H) variables are given in meters (m) while the mean sediment size (D_{50}) is given in mm, flow velocity (V) in m/s, and discharge (Q) is given in m³/s.

The Regime type equations proposed for Cauca River's tributaries is presented below (as bivariate expressions) as well as its statistical parameters for the best fit. Other expressions using non dimensional parameters as suggested by Parker (2004) are also exhibited next.

4.2.1 Bivariate type equations

$$\begin{aligned} W &= 1.339 D_{50}^{0.189} Q^{0.5} \qquad & R^2 &= 84\% \\ H &= 0.038 Q^{0.318} S_f^{\,0.36} \qquad & R^2 &= 74\% \\ V &= 0.788 D_{50}^{0.178} Q^{0.437} \qquad & R^2 &= 82\% \end{aligned} \tag{5}$$

The estimation of bed slope is the variable that offered bigger uncertainty on this work and also in all revised technical reports. The deviation of Cauca region-field data with respect to the estimated variables was high, for such a reason Parker's data was added to the field data for comparison purposes (Figure 2). Trying to find a representative variable related to slope, topographic (on site bottom slope) and cartographic (from large scale maps) slopes besides the friction slope were used. It was observed that friction and topographic slope did not present a clear tendency when compared to the estimated slope values, but the cartographic slope had some tendency excepting the rivers Molinos and Ovejas streams (circle in Figure 2). Then, dismissing these streams, an acceptable multiple correlation was found for the cartographic slope, bankfull discharge and mean diameter of bed sediments.

$$S = 0.0048 D_{50}^{1.07} Q^{\;0.46} \qquad R^2 = 74\% \tag{6}$$

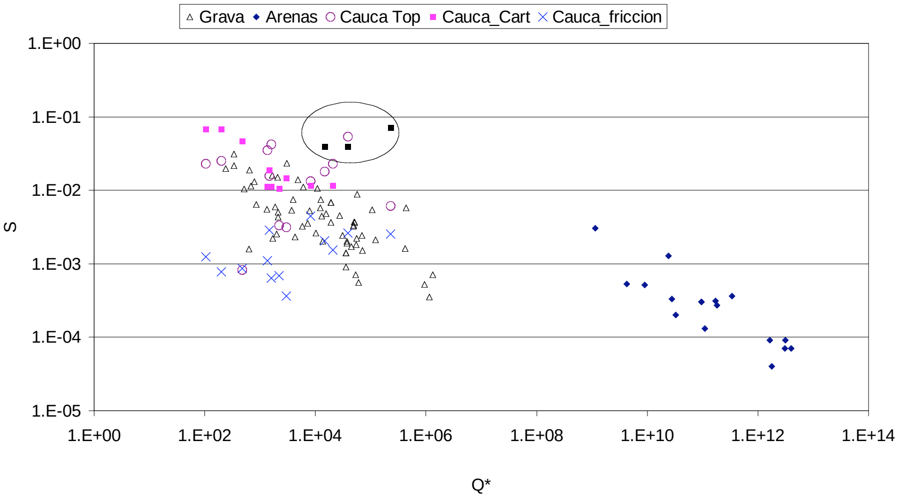
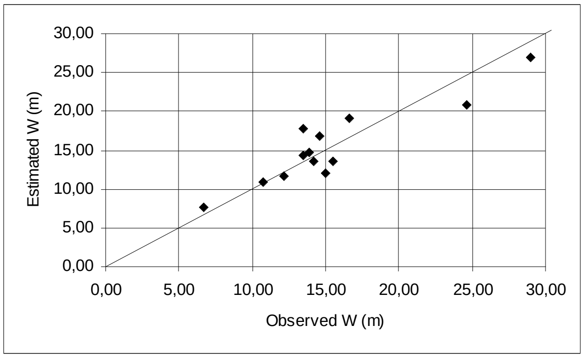
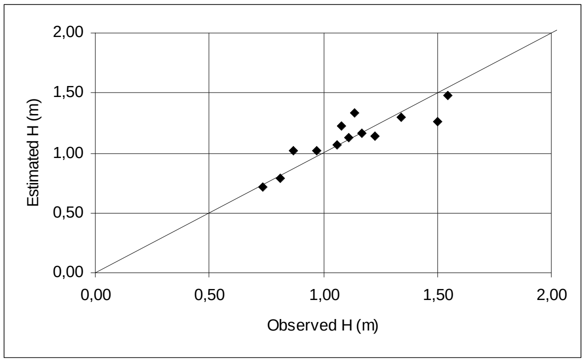


Figure 2 Relationship between Discharge and cartographic Slope for augmented set of data.

Figure 3 shows the relation between the observed variables and the estimated ones by means of the bivariate expressions. It is observed an appropriate adjustment for width, depth, and flow velocity while a great dispersion for the slope, as it was expected.





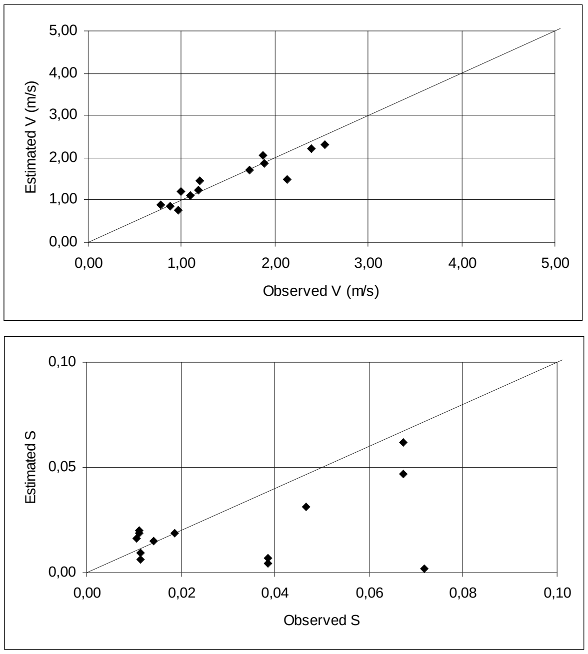


Figure 3 Relationships between observed and estimated variables by using the proposed bivariate expressions.

4.2.2 Parker type equations

As Parker proposed for better interpretations of data, dimensionless expressions for width, depth and flow discharge were analyzed and the resulting expressions are presented as equation 7:

$$W^* = 9.363 Q_b^{*0.358} \qquad H^* = 0.821 Q_b^{*0.34}$$
$$R^2 = 95.25\% \qquad R^2 = 86.4\% \tag{7}$$

To complete the variables defining the hydraulic geometry of the channel at bankfull discharge, the dimensionless velocity is presented as equation 8 and slope is presented as equation 9. The dimensionless velocity can be interpreted as a type of Froude number for sediment. For equation 9, data from Molinos and Ovejas streams was discarded since those points showed to be out of the range of the augmented data.

$$V^* = \frac{V}{\sqrt{gD_{50}}} \qquad V^* = 0.038 Q^{*0.465} \qquad R^2 = 97.2\% \tag{8}$$

$$S = 0.0463\, Q^{*0.434} \qquad R^2 = 74\% \tag{9}$$

The graphical relationship between the observed variables and Parker type estimated variables is shown in Figure 4.

Parker type dimensionless expressions give better estimates than those obtained by using regime bivariate type equations. In statistical analysis one can expect lower reliability when a great number of variables are involved for the multiple correlation. For this reason, the Parker type expressions are better recommended than bivariate proposed expressions.

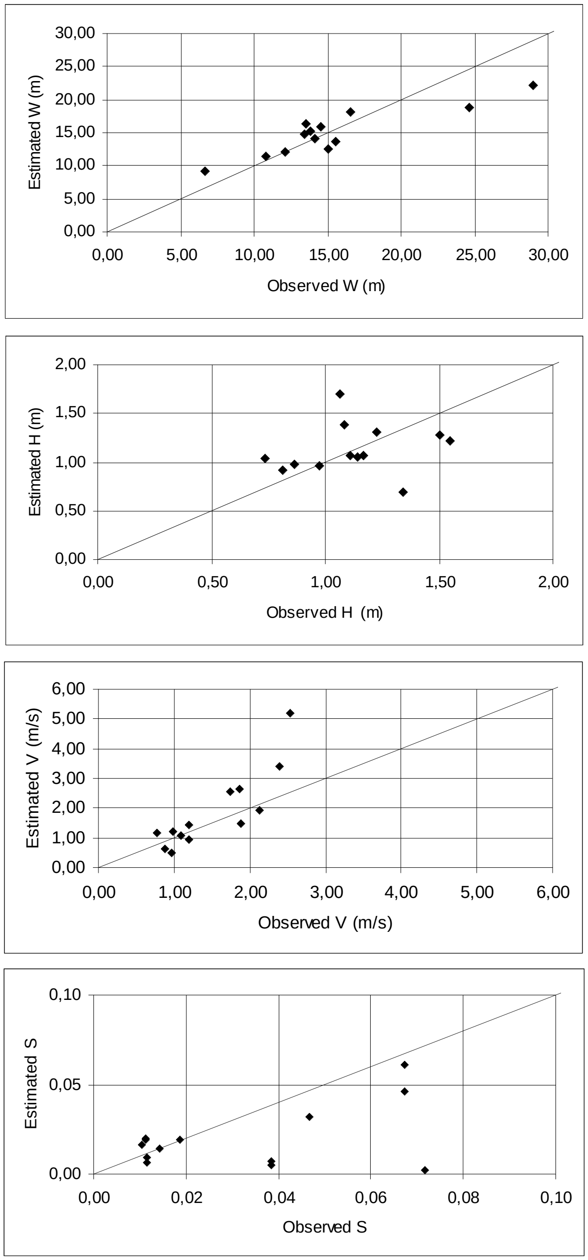


Figure 4 Variation of observed to estimated variables by using dimensionless Parker expressions.

5 Conclusions

There is no standardized methodology for measuring the friction slope especially for gravel-bed rivers where relative submergence (H/D_{50}) can be very low, the distribution of bed grains often is non regular, and the cross sectional area is very irregular for the presence of large size sediments. There must be a future investigation for defining the proper way to measure the slope to estimate the hydraulic geometry on gravel-bed rivers.

The field information was useful for establishing the Regime type relationships between the channel geometric variables, bankfull discharge and the characteristic sediment size (D_{50}) with correlation coefficients of 74-84%, that are considered high in fluvial geomorphology. To define the stable geometry, a rectangular channel or very wide one was considered to assume that the hydraulic radius is quite similar in magnitude to the flow depth. However, better correlation coefficients of the order of 90% were obtained for the relationships of dimensionless width and flow velocity variables as proposed by Parker (2004).

In statistical analysis, the bigger the number of variables, the lower the reliability of the estimates. This fact highly support the decision to recommend the Parker type equations instead of the bivariate Regime type equations obtained along this investigation on tributaries of Cauca River. Proposed expressions can be used for restorations studies, design of stable channels for mountain gravel rivers with mean sediment size greater than 2 mm.

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