How to get 3-D accuracy with a 2-D stream-aquifer interaction model (or "The Turning Resistance Factor in River Seepage")

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Abstract. Close to a river the traditional groundwater Dupuit assumption of horizontal flow does not hold. In order to treat the problem accurately one would need to use a 3-dimensional numerical model with small cells in the vicinity of the river. Presented here is an analytic approximate way to avoid this extra complication and still describe accurately the exchange flow between the stream and the aquifer.

1. Raison d'être for this investigation

In the vicinity of a river (or canal) reach the Dupuit-Forchheimer (D-F) assumption does **not** hold. In order to treat the problem accurately one would need to use a 3-dimensional model with small cells in the vicinity of the reach. Given the usually small horizontal spatial extent of the reach footprint, one may legitimately wonder if there might not be an **approximate yet adequate way to avoid this extra complication**?

2. Previous Work

To alleviate that difficulty it has been proposed to link a vertical analytical 2-dimensional solution to the numerical horizontal 2-dimensional model. A "reach transmissivity" is defined so that the exchange discharge Q (volume per unit time) would be given by the expression:

$$Q = \Gamma \Delta H \qquad (1)$$

where ΔH is the head difference between the reach and the aquifer at a finite distance from the reach. Eq. (1) is an *integrated form* of Darcy's law over a finite distance which accounts for the geometrical character of the flow path. How is Γ determined? It can be done analytically or numerically.

3. Analytical Approach

Figure 1 shows schematically such a flow path. Clearly in the vicinity of the reach bottom the flow is primarily vertical. Only as a distance x_{far} is reached does the flow become essentially horizontal. In the 1940's and 50's, it was assumed for simplicity, that the stream had no width and fully penetrated the aquifer. With the assumption of full penetration no resistance is accounted for the difficulty the flow encounters to change direction from a vertical to a horizontal one. The purpose of this investigation is to compare the results of an analytical approach for this added resistance "turning" factor with those obtained with: (a) the assumption of full penetration and (b) a Finite Difference grid.

4. A Finite Difference Dimensionless Conductance

The simplest grid one can conceive would be one that has one cell below the reach horizontal bed and of width the actual wetted perimeter. Figure 2 displays the configuration of that cell with center at point A where head is HA. Only half the reach

cross-section and half the cell is shown. Next to that cell to the right is another cell of width 2 Δx_{far} with center at point C where the head is HC. Beyond that cell further to the right the D-F assumption applies. Using Darcy's law and the principle of continuity the conductance is expressed as:

$$\Gamma_{FD} = \cfrac{1}{\left[\cfrac{e_B}{W_p} + \cfrac{W_p}{2e_B} + \cfrac{\Delta x_{far}}{e_B}\right]} \qquad (2)$$
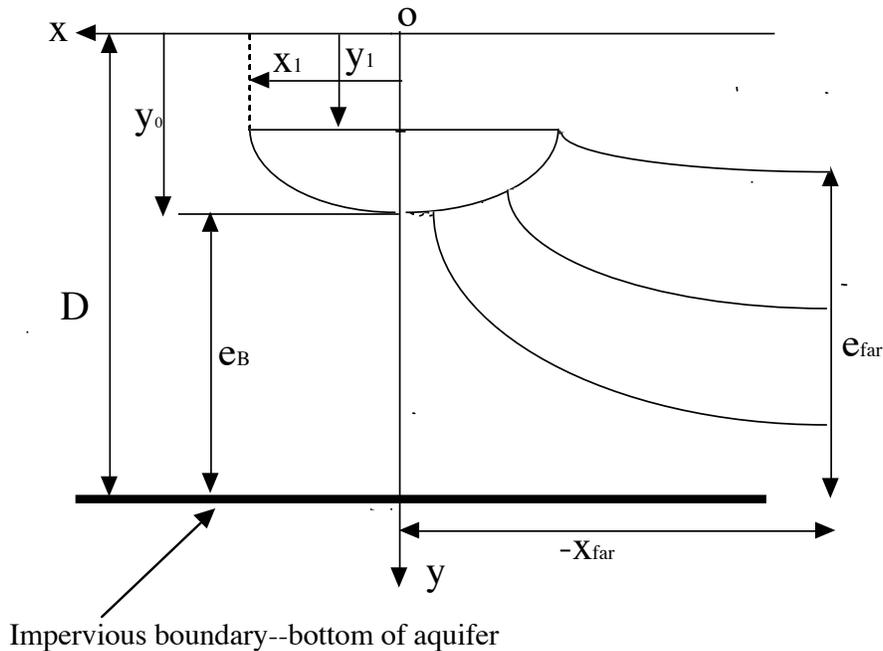

Figure 1. River or canal cross-section

The seepage flow from half the reach is then given as:

$$Q = KL\Gamma_{FD}(H_R - H_C) = KL\Gamma_{FD}\Delta H_{far} \qquad (3)$$

where ΔH_{far} is the head drop between the reach and the cell center located at a distance from the reach equal to Δx_{far}.

Figure 3 shows a comparison of the Analytical and the Finite Difference conductances, both expressed **relative** to that of Full Penetration. It is clear that the Finite Difference approach greatly underestimates the seepage. To get better accuracy with this approach one would have to have a grid that follows the contour of the cross-section and thus use a 3-dimensional model. However this is precisely what one wants to avoid.
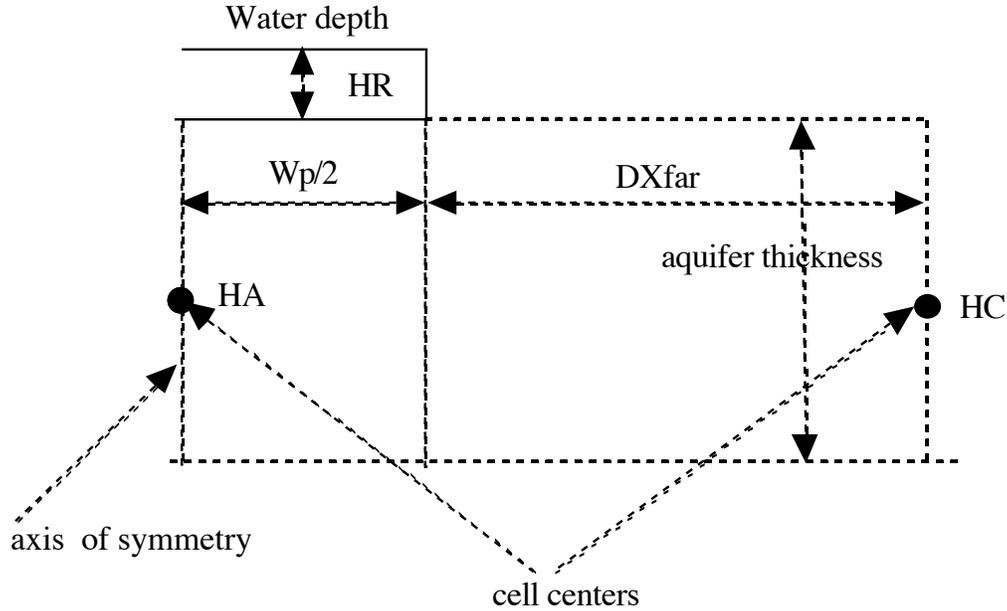
Figure 2. Finite Difference grid used to estimate stream-aquifer conductance

5. Practical Use of the Results of the Analytical Approach

Now how are the results shown in Figure 3 to be used? Take the ratio Γ_{AN}/Γ_{FP}, displayed in Figure 3, and call it Γ_r the "turning factor". It is the factor that needs to multiply the full penetration conductance to account for the added turning resistance to flow and for the difference in areas through which the flow moves. This is a function of the ratio WP/e and similarly is Δx_{far}, shown on Figure 4.

The exchange discharge from one side is then defined as:

$$Q = KL\Gamma_r\Gamma_{FP}\Delta H_{far} \qquad (4)$$

where ΔH_{far} is the horizontal head difference between the canal perimeter and the far section at $\Delta x_{far} = (x_{far} - B)$.

Consider a simple numerical example. Take a bottom aquifer thickness e_B of 10.0 m, a depth of 3.0, thus $\bar{e} = 11.50$, and a wetted perimeter of canal of 12.25 m.
Then WP/$\bar{e}$ = 12.25/11.50 = 1.17. From Figure 3 one obtains Γ_{FP} = 0.82 and Γ_r = 0.67.
Assume a hydraulic conductivity of 10.0 m/day, and a reach length of 1 km. Then the discharge from one side will be:
Q = 10x1000x0.67x0.82x ΔH_{far} = 5,494x ΔH_{far} cubic meters/day where ΔH_{far} is the drop of head, also measured in units of meters, over the distance $\Delta x_{far} = (x_{far} - B)$ =
1.23x 11.50= 14.1 meters (using Figure 4). Had one assumed full penetration the discharge would have been overestimated as 8,200x ΔH_{far} cubic meters/day.
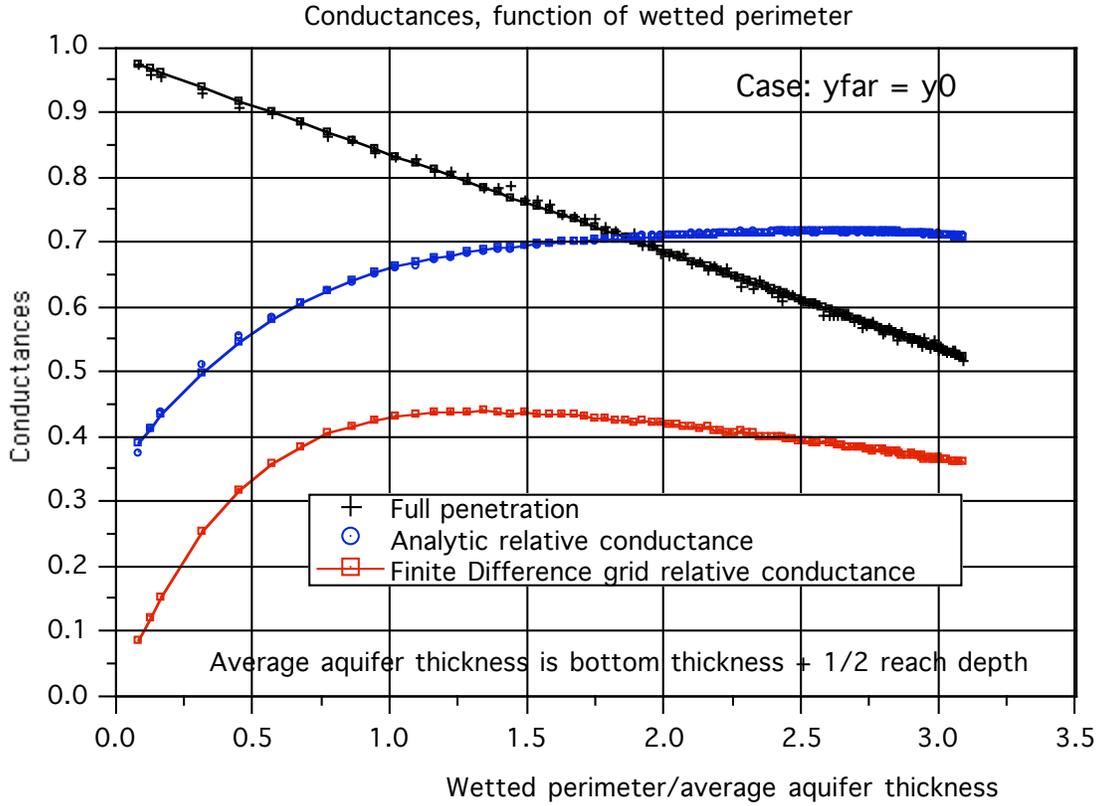
Conductances, function of wetted perimeter

Case: yfar = y0

Average aquifer thickness is bottom thickness + 1/2 reach depth

+   Full penetration
⊙   Analytic relative conductance
⊟   Finite Difference grid relative conductance

Conductances

Wetted perimeter/average aquifer thickness

Figure 3 Comparison of different estimations of stream aquifer conductance. Case: $y_{far} = y0$

Normalized water depth and distance to far section

Case: yfar = y0

⊙   Normalized reach water depth
⊡   Normalized distance to far section

Average aquifer thickness is bottom thickness + 1/2 reach depth

Reach water depth/average aquifer thickness

Normalized distance to far section

Wetted perimeter/average aquifer thickness

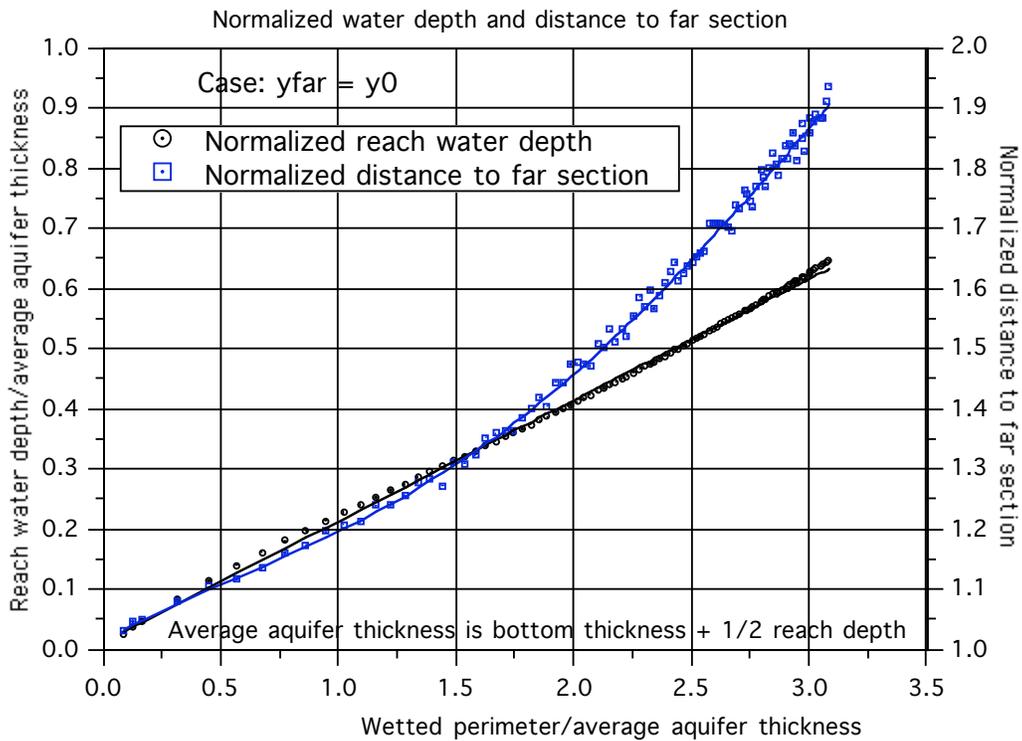**Figure 4** Normalized reach depth and distance to far section; Case: yfar = y0

One may rewrite Eq.(4) in the form:

$$Q = KL\Gamma_r\{WP/\bar{e}\}\frac{\bar{e}}{[\Delta x_{far}\{WP/\bar{e}\}]}\Delta H_{far} \quad (5)$$

where the dependence of the variables on the ratio WP/$\bar{e}$ has been shown explicitly through the braces {.}.

Had we used the Finite Difference estimation the discharge would have been:
Q = (0.44 /0.67)x5,494x ΔH_{far} = 3,608x ΔH_{far}

Application in Numerical Groundwater Modeling
With the proposed approach one can still formally assume full penetration provided that the discharge going into the cell with center at point C (see Figure 2) be given by Eq.(5).
Eq.(5) has the inconvenience that its direct application would require that distance from reach center to cell center be precisely x_{far}. Now typically in large scale groundwater models the cells are considerably wider than the reaches. Let then Δx_{RC} be the distance between the edge of the reach and the cell center and let $\Delta H_{RC} = H_R - H_C$ be the head drop between them. Again using Darcy's law and the principle of continuity one obtains for the conductance and the discharge in this case:

$$Q = KL\left\{\frac{1}{[1+(\frac{1}{\Gamma_r}-1)\frac{\Delta x_{far}}{\Delta x_{RC}}]}\right\}(\frac{\bar{e}}{\Delta x_{RC}})\Delta H_{RC} \quad (6)$$

In most groundwater models to calculate the river seepage the multiplier of the head difference would be treated as a constant. However it is clear from Eq.(6) that it would be very easy to account for the change in the reach transmissivity as a result of rise or fall of the stage in the reach since the relative conductance is a function of the wetted perimeter.
The great advantage of Eq.(6) is that it is no longer necessary to have a fine grid size in the vicinity of the reach to calculate accurately the seepage from the reach.

Case of clogging at the reach bottom
For simplicity let us define the relative dimensionless conductance in Eq. (6) as:

$$\Gamma_{rRC} = \left\{\frac{1}{[1+(\frac{1}{\Gamma_r}-1)\frac{\Delta x_{far}}{\Delta x_{RC}}]}\right\} \quad (7)$$

If there is clogging at the reach bottom with a hydraulic conductivity K_c over a thickness e_c then again simple algebra combined with mass conservation will yield a correction for the expression of the discharge:

$$Q = LK \left\{ \frac{1}{1+(\frac{K}{K_c})(\frac{e_c}{W_p})(\frac{\bar{e}}{\Delta x_{RC}})\Gamma_{rRC}} \right\} \Gamma_{rRC}(\frac{\bar{e}}{\Delta x_{RC}})\Delta H_{RC} \quad (8)$$

<u>Limitations of the Study</u>

One limitation, *among others*, is the fact that the results of Figures 3 and 4 depend on the shape of the river cross-section. Figure 5 shows the shape of the reach cross-section for a particular value of the ratio of wetted perimeter over average aquifer thickness. It is a reasonable shape.
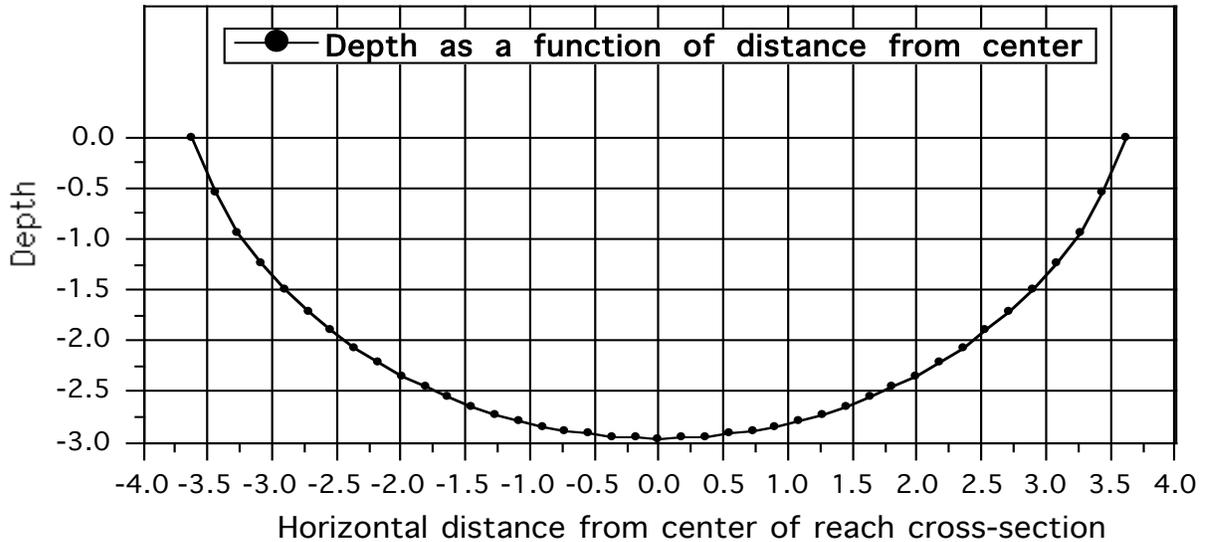
Depth/average aquifer thickness = 0.242
Wetted perimeter/average aquifer thickness = 0.984
Depth = 2.963   Half top width = 3.625

Figure 5. Cross section when depth of water is 2.42 units of length and aquifer thickness is 10

Conclusions

A potentially very practical tool has been suggested for a **numerically efficient and adequately accurate 2-dimensional alternative to a full detailed 3-dimensional modeling**. There is little need for a detailed grid description of the reach cross-section. In fact the full penetration assumption can be used provided that the flow across the reach vertical boundary be corrected by the turning factor. What has been quantitatively demonstrated is that neither the traditional assumption of full penetration nor the use of a typical Finite Difference grid model will provide adequately accurate results. However it must be emphasized that it is not necessary to proceed to a 3-dimensional model to improve accuracy but rather correct the manner in which the seepage discharge is calculated while conserving a typical Finite Difference grid.