Developing Rating Curves for Bedrock Step-Pool Rivers using Sparse Data

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Abstract. The objective of this study was to investigate the possibility of a generic rating curve with a small number of adjustable parameters for the restricted class of bedrock step-pool rivers. The motivation for this study arose from Utah Valley University's long-term program of groundwater development in the Trans-Mexican Volcanic Belt, which requires hydrographs for bedrock step-pool rivers near remote mountain villages. Based upon personal communications from USGS state hydrologists and Google Earth satellite images and aerial photos, 71 USGS active or historic gaging sites were identified as bedrock step-pool rivers, involving 403 rating numbers and 13,409 gage height vs. discharge measurements. For comparison, 71 gaging sites were chosen randomly, involving 268 rating numbers and 7617 gage height vs. discharge measurements. As the distributions of gage heights and discharges were a good fit to a lognormal distribution, the data were normalized by computing the Z-scores of the logarithms of discharge and gage height, in which the average and standard deviation were computed separately for each rating number. The linear relationships between Z-scores of gage height (y-axis) and discharge (x-axis) were surprisingly similar for bedrock step-pool and random rivers. For bedrock step-pool rivers, the slope was 0.8829 with $R^2 = 0.78$, while for random rivers, the slope was 0.8467 with $R^2 = 0.72$. Numerical experiments on generating rating curves based on two random measurements for each rating number showed that 50% of all observed discharges deviated from predicted discharges by less than 31.0% for bedrock step-pool rivers and 27.3% for random rivers.

1. Introduction

A rating curve is an empirical relationship between stream discharge and gage height that is used to derive a hydrograph from a record of gage height. The conventional practice is to develop a rating curve from multiple paired measurements of discharge and gage height. The objective of this study was to investigate whether, for a sufficiently restricted class of rivers, the vast database of existing gage height vs. discharge measurements might be used to generate a generic rating curve with a small number of adjustable parameters so as to reduce the number of measurements required for development of a rating curve at a particular site. The chosen restricted class of rivers was that of bedrock step-pool rivers, which are rivers that flow along a stair-like course that is cut into bedrock (Hayward 1980; Wohl and Grodek 1994) (see Fig. 1). The motivation for this study arose from Utah Valley University's long-term program of groundwater development in the Trans-Mexican Volcanic Belt, which requires hydrographs for bedrock step-pool rivers near remote mountain villages.

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Figure 1. The South Fork of Mill Creek is a typical bedrock step-pool river near Moab in southern Utah.

Most recent work on rating curves has involved the development of rating curves based on channel geometry without any gage height vs. discharge measurements (Bailey 1966; Drus 1982; Davidian 1984; Jarrett and Malde 1987; Di Giammarco et al. 1998; Wilson et al. 2002; Gergov and Karagiozova 2003; Kean and Smith 2005, 2010; Szilagyi et al. 2005; Perumal et al. 2007, 2010; Smith et al. 2010), the use of parameters in addition to gage height to predict discharge (Sahoo and Ray 2006; Sha 2007), the methodology and accuracy of developing rating curves based on gage height vs. discharge measurements (Dymond and Christian 1982; DeGagne et al. 1996; Clarke 1999; Peterson-Overleir 2004; Peterson-Overleir and Reitan 2005; Jalbert et al. 2011) and the use of remote discharge measurements to develop rating curves (Birkhead and James 1998). We are not aware of any other work on reducing the number of gage height vs. discharge measurements required to develop a rating curve based on the statistics of the existing gage height vs. discharge database.

2. Methods

As the USGS has no systematic database of site descriptions or photos of stream gaging sites, the primary source of information in identifying gaging sites on bedrock step-pool rivers was the personal knowledge of the USGS state hydrologists in all 50 states. This personal knowledge was supplemented by Google Earth satellite images and aerial photos (see Fig. 2), Google Earth location photos, topographic relief and slope, and surrounding geologic features (e.g., exposed bedrock formations that the river cut through). The Google Earth satellite images and aerial photos were regarded as the least diagnostic and in no case was a bedrock step-pool river identified by Google Earth satellite images or aerial photos alone. A total of 71 active or historic stream gaging sites with records of gage height vs. discharge measurements were identified as bedrock step-pool rivers (see Table 1). These 71 sites involved 403 rating numbers and 13,409 paired gage height – discharge measurements (USGS 2012). (A rating number is the set of measurements used to develop a rating curve. The USGS practice is that, when a measured discharge differs from the

discharge predicted by the rating curve by more than 10%, a new set of measurements is begun to develop a new rating curve). For comparison, a random number generator was used to select 71 random stream gaging sites involving 268 rating numbers and 7617 paired measurements.

Figure 2. This Google Earth aerial photo is of a section of the Wainiha River of Kauai, Hawaii, just northwest of gaging station 16108000 (see Table 1). This photo shows a bedrock step-pool section of river (left) and a calm section (right). Bedrock step-pool rivers were not identified by Google Earth satellite images and aerial photos alone, but only in conjunction with location photos and personal communication from USGS state hydrologists.

For both bedrock step-pool rivers and random rivers, the cumulative percentages of both gage heights and discharges were plotted against the cumulative percentages of the equivalent normal and lognormal distributions, based on the averages and standard deviations of all gage heights and all discharges, respectively. In no case was the distribution even remotely close to a normal distribution (see Figs. 3a-b). On the other hand, the distribution of discharges for bedrock step-pool rivers and the distributions of both gage heights and discharges for random rivers were nearly indistinguishable from the equivalent lognormal distribution (see Figs. 4a-b). The distribution of gage heights for bedrock step-pool rivers was reasonably close to a lognormal distribution (see Fig. 4a). On that basis, all least-squares regression analysis was carried out on the logarithms of values, rather than the values themselves.
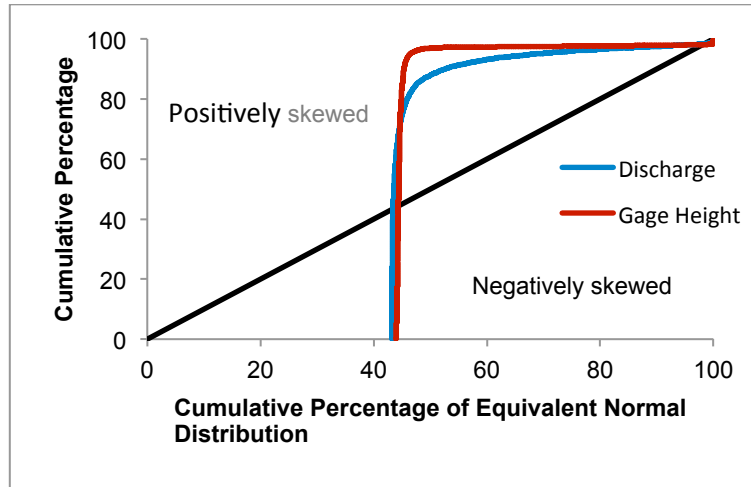
For each measurement of gage height and discharge, the Z-scores were computed as

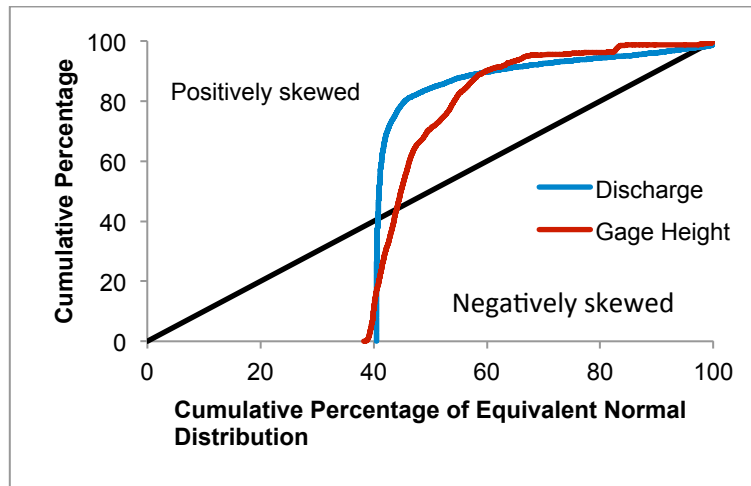$$Z_{\ln GH} = \frac{\ln GH - \overline{\ln GH}}{\sigma_{\ln GH}} \tag{1}$$

$$Z_{\ln Q} = \frac{\ln Q - \overline{\ln Q}}{\sigma_{\ln Q}} \tag{2}$$

where GH is gage height, Q is discharge, σ is standard deviation and an overbar indicates the mean. For each measurement, the mean and standard deviation used to compute the Z-score was the mean and standard deviation of the set of measurements with the same rating

number for a given gaging site, so that, essentially, each rating number was regarded as a distinct gaging site.

Figure 3a. For bedrock step-pool rivers, the distributions of discharges and gage heights are not remotely close to a normal distribution.

Figure 3b. For randomly chosen rivers, the distributions of discharges and gage heights are not remotely close to a normal distribution.
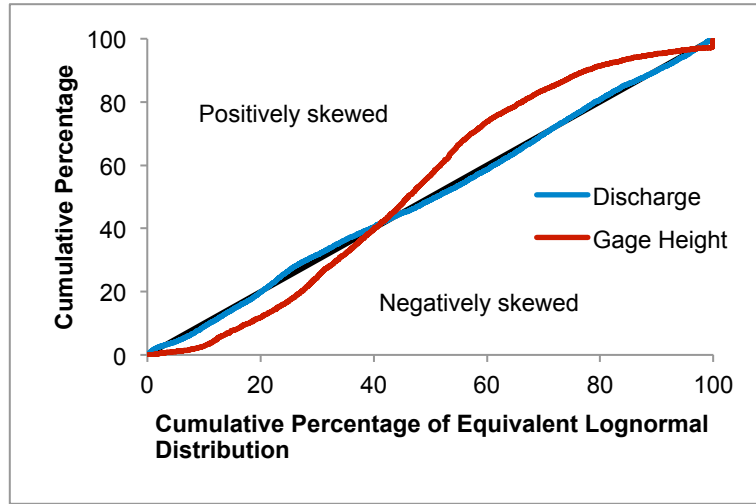
3. Results

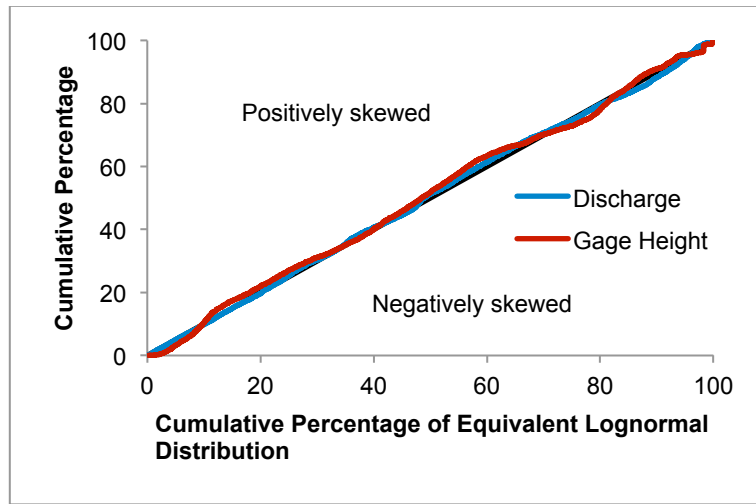The generic relationship between the Z-scores of the logarithms of gage height and discharge is

$$Z_{\ln GH} = aZ_{\ln Q} + b \qquad (3)$$

where $a = 0.8828$ for bedrock step-pool rivers, $a = 0.8467$ for rivers chosen randomly and $b = 0$ in both cases. The fit is somewhat better for bedrock step-pool rivers ($R^2 = 0.78$) than for random rivers ($R^2 = 0.72$) (see Figs. 5-6). Note that there is no justification for fitting a curve any more complex than a straight line. The isolated points at the low and high discharges are a very small fraction of the total number of points and exert essentially no influence on the fit of the curve. It is quite surprising, first, that the generic rating curve for

randomly chosen rivers is so good and, second, that the generic rating curve for deliberately selected bedrock step-pool rivers is not significantly better.

Figure 4a. For bedrock step-pool rivers, the distribution of discharges is indistinguishable from a lognormal distribution, while the distribution of gage heights is reasonably close to a lognormal distribution.
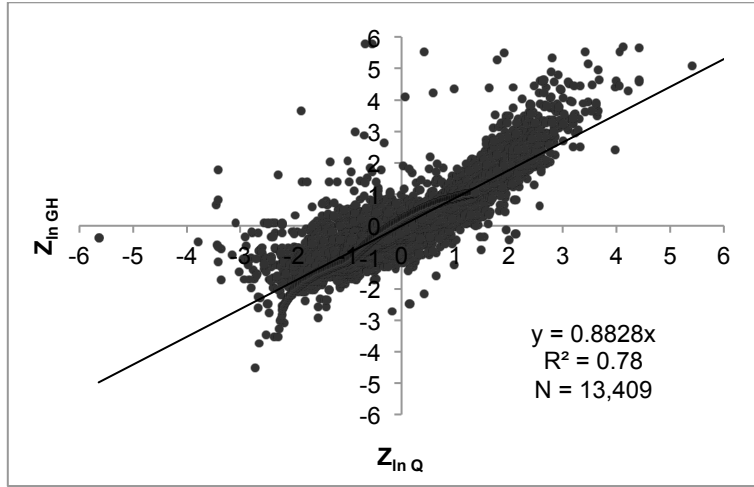

Figure 4b. For randomly chosen rivers, the distributions of both discharges and gage heights are indistinguishable from a lognormal distribution.
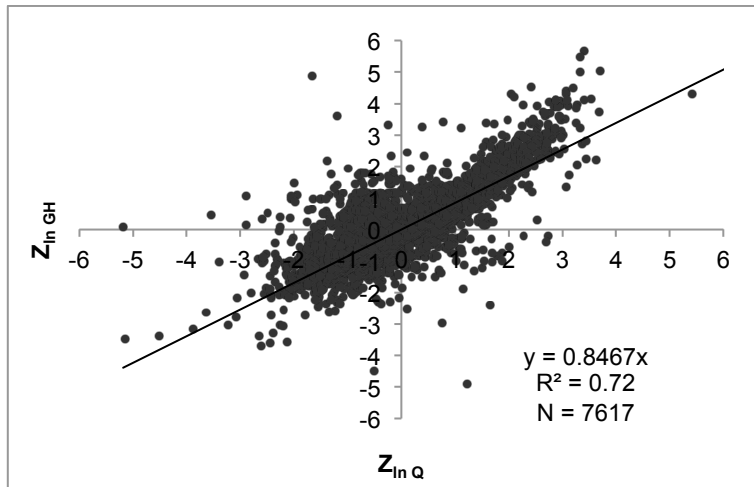
4. Discussion

The empirical relationship in Eq. (4) seems to require the four adjustable parameters $\overline{\ln Q}$, $\sigma_{\ln Q}$, $\overline{\ln GH}$, and $\sigma_{\ln GH}$. However, the last two parameters can simply be determined from the record of gage heights without any simultaneous measurements of discharge. In fact, Utah Valley University's program of groundwater development in the Trans-Mexican Volcanic Belt involves organizing local residents to carry out daily measurements of stream stage using meter sticks. Given that $\overline{\ln GH}$ and $\sigma_{\ln GH}$ are known parameters, the two adjustable parameters $\overline{\ln Q}$ and $\sigma_{\ln Q}$ can be determined from any two paired measurements of gage height and discharge (GH_1, Q_1) and (GH_2, Q_2) by combining Eqs. (1) - (3) to yield

$$\sigma_{\ln Q} = a\sigma_{\ln GH} \frac{\ln(Q_1/Q_2)}{\ln(GH_1/GH_2)} \qquad (4)$$

$$\overline{\ln Q} = \ln Q_1 - \frac{\ln(Q_1/Q_2)}{\ln(GH_1/GH_2)}(\ln GH_1 - \overline{\ln GH}). \qquad (5)$$

Figure 5. For bedrock step-pool rivers, there is a good linear relationship ($R^2 = 0.78$) between the Z-score of the logarithm of discharge ($Z_{\ln Q}$) and the Z-score of the logarithm of gage height ($Z_{\ln GH}$).

Figure 6. For randomly chosen rivers, there is a good linear relationship ($R^2 = 0.72$) between the Z-score of the logarithm of discharge ($Z_{\ln Q}$) and the Z-score of the logarithm of gage height ($Z_{\ln GH}$). The generic rating curves for bedrock step-pool rivers and randomly chosen rivers are very similar.

Numerical experiments to evaluate the accuracy of the above procedure in predicting discharge from gage height were carried out by using a random number generator to choose two paired measurements of gage height and discharge for each rating number, using Eqs. (4) – (5) to determine the adjustable parameters $\overline{\ln Q}$ and $\sigma_{\ln Q}$ for each rating number, using Eqs. (1) – (2) to determine Z-scores, and then using Eq. (3) to predict discharge from each of 13,409 measurements of gage height for bedrock step-pool rivers

and 7617 measurements of gage height for random rivers, with the parameter a chosen as $a = 0.8828$ for bedrock step-pool rivers and $a = 0.8467$ for rivers chosen randomly. The percent deviation was calculated as

$$\text{Percent Deviation} = 100 \times \left| \frac{Q_{pred} - Q_{obs}}{Q_{pred}} \right| \qquad (6)$$

where Q_{pred} is predicted discharge and Q_{obs} is observed discharge. Based on the numerical experiments, 50% of all observed discharges deviated from predicted discharges by less than 31.0% for bedrock step-pool rivers and 27.3% for random rivers. The percentage of observed discharges that differed from predicted discharges by less than a factor of two (percent deviation = 100% when $Q_{obs} = 2Q_{pred}$) was 86.5% for bedrock step-pool rivers and 89.0% for random rivers. The percentage of observed discharges that differed from predicted discharges by less than an order of magnitude (percent deviation = 900% when $Q_{obs} = 100Q_{pred}$) was 98.1% for bedrock step-pool rivers and 99.2% for random rivers (see Fig. 7). The procedure performed slightly better for rivers chosen randomly than for deliberately chosen bedrock step-pool rivers.

Figure 7. Based on numerical experiments, 50% of all observed discharges will deviate from predicted discharges by less than 31.0% for bedrock step-pool rivers and 27.3% for random rivers. The percentage of observed discharges that will differ from predicted discharges by less than a factor of two (percent deviation = 100%) will be 86.5% for bedrock step-pool rivers and 89.0% for random rivers. The percentage of observed discharges that will differ from predicted discharges by less than an order of magnitude (percent deviation = 900%) will be 98.1% for bedrock step-pool rivers and 99.2% for random rivers.

5. Conclusions

The first surprising result of this study was that there is a reasonably good generic rating curve for all rivers, which does not seem to have been previously reported. This generic rating curve could be used in any situation for which errors in predicted discharge of less than about 30% are acceptable. The second surprising result was that deliberating choosing rivers of a given type, in this case, bedrock step-pool rivers, did not produce a generic rating curve that was significantly different from or more accurate than the generic rating curve for random rivers. The question as to how to do choose a sufficiently

restricted class of rivers such that those rivers will have an accurate generic rating curve remains open.

Appendix: Catalog of Bedrock Step-Pool Rivers.

Table 1. Catalog of bedrock step-pool rivers with USGS gaging stations.

State	Site Number[*]	Site Name
AL	02398950	WEST FORK LITTLE R AT DESOTO PARK NR FORT PAYNE AL
AL	02399200	LITTLE RIVER NEAR BLUE POND AL
AZ	09505200	WET BEAVER CREEK NEAR RIMROCK, AZ
CA	11433040[*]	PILOT C BL MUTTON CANYON NR GEORGETOWN CA
CA	11201850[*]	NF OF MF TULE R BL DOYLE SP DIV NR SPRINGVILLE
CA	11202000[*]	NF OF MF TULE R NR SPRINGVILLE(RIVER ONLY) CA
CA	11206500[*]	MF KAWEAH R NR POTWISHA CAMP(RIVER ONLY) CA
CA	11208000[*]	MARBLE F KAWEAH R (R ONLY) A POTWISHA CAMP CA
CA	11208600[*]	KAWEAH R BL CONDUIT NO. 2 NR HAMMOND CA
CA	11208730[*]	EF KAWEAH R NR THREE RIVERS CA
CA	11209900[*]	KAWEAH R A THREE RIVERS CA
CA	11274790	TUOLUMNE R A GRAND CYN OF TUOLUMNE AB HETCH HETCHY
CA	11281000[*]	SF TUOLUMNE R NR OAKLAND RECREATION CAMP CA
CA	11283250[*]	CLAVEY R NR LONG BARN CA
CA	11293650[*]	NF STANISLAUS R A CAMP WOLFEBORO NR BIG MEADOWS CA
HI	16108000	Wainiha River nr Hanalei, Kauai, HI
HI	16501200	Oheo Gulch at dam near Kipahulu, Maui, HI
HI	16552800	Waikamoi Str abv Kula Pl intake nr Olinda, Maui,HI
HI	16704000	Wailuku River at Piihonua, HI
HI	16770500	Paauau Gulch at Pahala, HI
ME	01021200	Dennys River at Dennysville, Maine
ME	01030500	Mattawamkeag River near Mattawamkeag, Maine
ME	01031450	Kingsbury Stream at Abbot Village, Maine
ME	01052500	Diamond River near Wentworth Location, NH
ME	01055000	Swift River near Roxbury, Maine
MI	04043150	SILVER RIVER NEAR L'ANSE, MI
MN	04010500	PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN
MN	05336700	KETTLE RIVER BELOW SANDSTONE, MN
MN	05129115	VERMILION RIVER NR CRANE LAKE, MN
MN	05126210	SOUTH KAWISHIWI R ABV WHITE IRON LAKE NR ELY, MN
NV	10324500	ROCK CK NR BATTLE MOUNTAIN, NV
NV	10310400	DAGGETT CK NR GENOA, NV
NH	01064300[*]	ELLIS RIVER NEAR JACKSON, NH
NH	01064400[*]	LUCY BROOK NEAR NORTH CONWAY, NH
NH	01075800[*]	STEVENS BROOK NEAR WENTWORTH, NH
NJ	01461300	Wickecheoke Creek at Stockton NJ
NC	03441440[*]	LITTLE RIVER AB HIGH FALLS NEAR CEDAR MT, NC

OH	04197170	Rock Creek at Tiffin OH
OH	04197137	Sandusky River at Tiffin OH
OH	04196800	Tymochtee Creek at Crawford OH
OK	07332390	Blue River near Connerville, OK
OR	14101500[*]	WHITE RIVER BELOW TYGH VALLEY,OREG
PA	04213152	Walnut Creek Upstream Pool, near Erie, PA
PA	04213075	Brandy Run near Girard, PA
TX	07311600	N Wichita Rv nr Paducah, TX
TX	08049580	Mountain Ck nr Venus, TX
TX	08055000	Denton Ck nr Grapevine, TX
TX	08084000	Clear Fk Brazos Rv at Nugent, TX
TX	08091500	Paluxy Rv at Glen Rose, TX
TX	08095000	N Bosque Rv nr Clifton, TX
TX	08101000	Cowhouse Ck at Pidcoke, TX
TX	08103800	Lampasas Rv nr Kempner, TX
TX	08105100[*]	Berry Ck nr Georgetown, TX
TX	08143600	Pecan Bayou nr Mullin, TX
TX	08144600	San Saba Rv nr Brady, TX
TX	08168797	Dry Comal Ck at Loop 337 nr New Braunfels, TX
TX	08181400	Helotes Ck at Helotes, TX
TX	08183500	San Antonio Rv nr Falls City, TX
TX	08185000	Cibolo Ck at Selma, TX
TX	08192000	Nueces Rv bl Uvalde, TX
TX	08197500	Frio Rv bl Dry Frio Rv nr Uvalde, TX
TX	08200977	Middle Verde Ck at SH 173 nr Bandera, TX
TX	08201500	Seco Ck at Miller Rh nr Utopia, TX
TX	08202700	Seco Ck at Rowe Rh nr D'Hanis, TX
TX	08206600	Frio Rv at Tilden, TX
TX	08211200	Nueces Rv at Bluntzer, TX
UT	09183000[*]	COURTHOUSE WASH NEAR MOAB, UTAH
UT	09184000[*]	MILL CREEK NEAR MOAB, UT
WA	12448000	CHEWUCH RIVER AT WINTHROP, WA
WA	12104500	GREEN RIVER NEAR LESTER, WA
WA	12185300	ELLIOTT CR AT GOAT LAKE OUTLET NR MONTE CRISTO, WA

[*] indicates station is longer active but historical records are available

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