A Conceptual Framework for the Use of Machine Learning for the Synthesis of Stream Discharge – Gage Height Rating Curves

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Abstract. The objective of this research is to use machine learning for the synthesis of stream discharge – gage height rating curves from easily measurable hydrogeologic parameters. A machine learning algorithm would require as input a compilation of relevant hydrogeologic parameters for each gaging station. Since such a compilation does not yet exist, the first step has been to create a conceptual framework that identifies the relevant hydrogeologic parameters that would need to be compiled. Frequent reverse flow or flood waves preclude the existence of a rating curve (unique relationship between gage height and discharge). If a rating curve exists, then a stable channel has a power-law rating curve. Deviations from the power-law curve result from deposition (power-starvation) or scouring (sediment-starvation), which could occur at the high or low range of discharge or both. The eight types of deviation (including no deviation) from the power-law curve can be regarded as eight functional forms of rating curves, which can be represented as lines, parabolas or cubic polynomials on plots of the Z-scores of the logarithms of gage height and discharge. Rating curves can be classified into the eight types based on the hydrogeologic criteria of (1) stream slope (2) relative erodibility of the stream banks (3) distance to the nearest upstream and downstream confluences with relatively significant discharge. USGS gaging stations in Utah were chosen randomly until each of the eight types of rating curves was found. The first example of each type was shown to be consistent with the corresponding hydrogeologic criteria.

1. Introduction

A rating curve is an empirical relationship between gage height and stream discharge that is used to derive a hydrograph from a record of gage height. The development of a rating curve requires multiple simultaneous field measurements of gage height and discharge over a wide range of discharge values, so that rating curve development is a major manpower expense for state and federal agencies that monitor stream discharge. The objective of this research is to use machine learning for the synthesis of stream discharge – gage height rating curves from easily measurable hydrogeologic parameters. A machine learning algorithm would require as input the existing gage height vs. discharge database for a large number of gaging stations as well as a compilation of relevant hydrogeologic parameters for those gaging stations that would be used to build a model for the prediction of rating curves from hydrogeologic parameters. Since such a compilation does not yet exist, the first step has been to create a conceptual framework that identifies the relevant hydrogeologic parameters that would need to be compiled. This research has been motivated by a Utah Valley University project on the installation of small-scale compressed-air hydropower stations in rural Haiti. Hydropower requires a record of stream discharge and rating curve synthesis is needed to convert into stream discharge the daily measurements of gage height that are being made manually by local residents.

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Most recent work on rating curves has involved the development of rating curves based on channel geometry without any gage height vs. discharge measurements (Kean and Smith 2005; Szilagyi et al. 2005; Perumal et al. 2007, 2010), the use of parameters in addition to gage height to predict discharge (Sahoo and Ray 2006; Weijs et al. 2013), the methodology and accuracy of developing rating curves based on gage height vs. discharge measurements (Morlot et al. 2014; Singh et al. 2014; Coxon et al. 2015) and the use of remote discharge measurements to develop rating curves (Birkhead and James 1998). We are not aware of any other previous work besides that of the authors (Stuart and Emerman 2012; Rundall et al. 2015) on the development of rating curves based on the statistics of the existing rating curve database.

2. Analysis

Rating curves can be divided into eight types, each of which has a mathematical, a hydraulic, and a hydrogeologic description (see Table 1). The hydrogeologic descriptions specify the hydrogeologic parameters that would be required for the synthesis of rating curves. The first type of rating curve (Type 1) is the one that does not exist because there is a non-unique relationship between gage height and discharge. The primary sources of non-unique relationships are unstable channels, reverse flow from downstream tributaries, and unsteady flow, which could result either from tides or flood waves (Kennedy 1984). Based on the 209 USGS gaging stations in Utah with a useable history of simultaneous measurements of gage height and discharge, Rundall et al. (2015) showed that extreme variation in gage height without accompanying variation in discharge (defined as a linear fit with $R^2 < 0.6$) occurred for gaging stations close to either the nearest upstream confluence or downstream confluence (defined as 10% of the distance along the stream between the upstream and downstream confluences).

The assumption of this analysis is that a stable channel has a power-law rating curve and that deviations from the power-law curve result from deposition (power-starvation) or scouring (sediment starvation) (see Fig. 1a). Deposition causes measurements to plot above the power-law curve, while scouring causes measurements to plot below the power-law curve, using the conventional plot of gage height as the y-axis and discharge as the x-axis (see Fig. 1a). A power-law rating curve for a stable channel is a consequence of either the Manning Equation or the Chézy Equation or of any resistance equation that assumes that the resisting force is a power-law function of the velocity (Dingman 2009). A power-law curve on an arithmetic scale is a straight line on a plot of the logarithmic Z-scores

$$\mathrm{Z}_{\ln GH} = \frac{\ln GH - \overline{\ln GH}}{\sigma_{\ln GH}} \qquad (1)$$

$$\mathrm{Z}_{\ln Q} = \frac{\ln Q - \overline{\ln Q}}{\sigma_{\ln Q}} \qquad (2)$$

where GH is gage height, Q is discharge, σ is standard deviation and an overbar indicates the mean (see Fig. 1a). A logarithmic Z-score plot is a preferable mathematical description since numerous studies (e.g., Stuart and Emerman 2012) have shown that both gage height and discharge data are a better fit to a lognormal than a normal distribution, so that regression modeling should be carried out on the logarithms of values. Moreover, non-dimensional variables allow for the comparison of streams of very different sizes. A linear fit on a logarithmic Z-score plot (Type 2 rating curve) should be expected whenever there is a balance between stream power and sediment supply, in other words, when the stream has moderate slope and the stream banks have moderate erodibility (see Table 1).

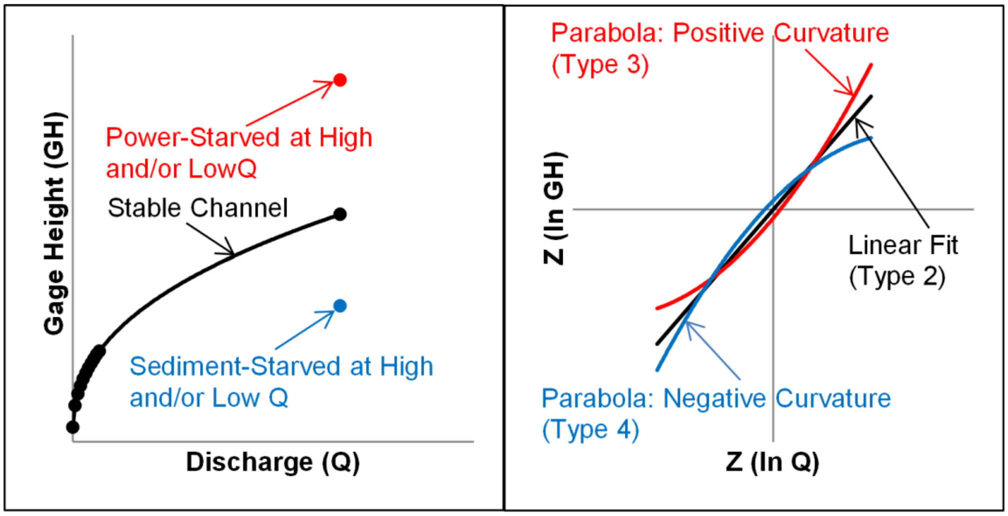

Figure 1a. In a stable channel, a plot of gage height as a function of discharge will be a power-law curve, while a plot of the Z-score of the logarithm of gage height as a function of the Z-score of the logarithm of discharge will be a straight line (Type 2 rating curve). If deposition (power-starvation) occurs at high and/or low discharge, the Z-score logarithmic plot will be a parabola with positive curvature (Type 3 rating curve). If scouring (sediment-starvation) occurs at high and/or low discharge, the Z-score logarithmic plot will be a parabola with negative curvature (Type 4 rating curve). Deviations from the stable channel curve at low discharge cannot be seen at the scale of the left-hand diagram.

The simplest deviations from the stable channel are that either deposition (power-starvation) or scouring (sediment-starvation) could occur at high and/or low discharge. If deposition occurs, then the Z-score logarithmic plot will be a parabola with positive curvature (Type 3 rating curve, see Fig. 1a). If scouring occurs, the Z-score logarithmic plot will be a parabola with negative curvature (Type 4 rating curve, see Fig. 1a). A parabolic fit with positive curvature would be expected if the stream has low slope and the stream banks are unconsolidated materials. A parabolic fit would be expected if the stream has high slope with bedrock stream banks (see Table 1).

More complex deviations from the stable channel occur when deposition and scouring occur at opposite ends of the range of discharges. If the gaging site is overall power-starved, then the best-fit parabola will still have positive curvature, which is consistent with low slope and unconsolidated stream banks. If scouring occurs at high discharge and deposition occurs at low discharge, then the best-fit cubic polynomial will have a negative cubic coefficient (Type 3.1 rating curve, see Fig. 1b). This is consistent with fine grain sizes, which could be mobilized even at low discharge (see Table 1). On the other hand, if deposition occurs at high discharge and scouring occurs at low discharge, then the best-fit cubic polynomial will have a positive cubic coefficient (Type 3.2 rating curve, see Fig. 1b). This is consistent with coarse grain sizes, which could be mobilized only at high discharge (see Table 1). On the above basis, a moderate grain size should predict a Type 3 rating curve, in which the parabolic fit is not improved by considering a cubic polynomial (see Fig. 1a, Table 1).

Table 1. Classification of stream discharge – gage height rating curves.

Type	Characteristics		
	Mathematical[1]	Hydraulic	Hydrogeologic
1	No linear, parabolic or cubic fit ($R^2 < 0.6$)	Reverse flow or flood waves	Close to upstream or downstream confluence
2	Parabolic no better than linear fit	Stable channel, balance of power and sediment supply	Moderate slope, moderate stream bank erodibility
3	Parabolic fit better than linear, parabola has positive curvature, cubic no better than parabolic fit	Power-starved at low and/or high discharge	Low slope, unconsolidated stream banks, moderate grain size
3.1	Parabolic fit better than linear, parabola has positive curvature, cubic better than parabolic fit, negative cubic coefficient	Overall power-starved, power-starved at low discharge and sediment-starved at high discharge	Low slope, unconsolidated stream banks, fine grain size
3.2	Parabolic fit better than linear, parabola has positive curvature, cubic better than parabolic fit, positive cubic coefficient	Overall power-starved, sediment-starved at low discharge and power-starved at high discharge	Low slope, unconsolidated stream banks, coarse grain size
4	Parabolic fit better than linear, parabola has negative curvature, cubic no better than parabolic fit	Sediment-starved at low and/or high discharge	High slope, bedrock stream banks, moderate erodibility
4.1	Parabolic fit better than linear, parabola has negative curvature, cubic better than parabolic fit, negative cubic coefficient	Overall sediment-starved, power-starved at low discharge and sediment-starved at high discharge	High slope, bedrock stream banks, bedrock susceptible to erosion
4.2	Parabolic fit better than linear, parabola has negative curvature, cubic better than parabolic fit, positive cubic coefficient	Overall power-starved, sediment-starved at low discharge and power-starved at high discharge	High slope, bedrock stream banks, bedrock resistant to erosion

[1]Mathematical characteristics refer to plots of the Z-score of ln GH (y-axis) vs. the Z-score of ln Q (x-axis), where GH is gage height and Q is discharge.

Similar arguments can be used to develop the last two types of rating curves. If the gaging site is overall sediment-starved, then the best-fit parabola will still have negative curvature, which is consistent with high slope and bedrock stream banks. If scouring occurs at high discharge and deposition occurs at low discharge, then the best-fit cubic polynomial will have a negative cubic coefficient (Type 4.1 rating curve, see Fig. 1c). This is consistent with bedrock that is susceptible to erosion, which could be mobilized even at low discharge (see Table 1). On the other hand, if deposition occurs at high discharge and scouring occurs at low discharge, then the best-fit cubic polynomial will have a positive cubic coefficient (Type 4.2 rating curve, see Fig. 1c). This is consistent with bedrock that is resistant to erosion, which could be mobilized only at high discharge (see Table 1). On the above basis, bedrock with moderate erodibility should predict a Type 4 rating curve, in which the parabolic fit is not improved by considering a cubic polynomial (see Fig. 1c, Table 1). In summary, the hydrogeologic criteria that are required for the placement of gaging stations into rating curve types are (1) stream slope (2) stream bank erodibility (3) distance to the nearest upstream or downstream confluence. The above criteria do not act independently. For example, a Type 3 rating curve might result from a combination of a very low slope and stream banks composed of highly erodible fine-grained unconsolidated material (see Table 1).

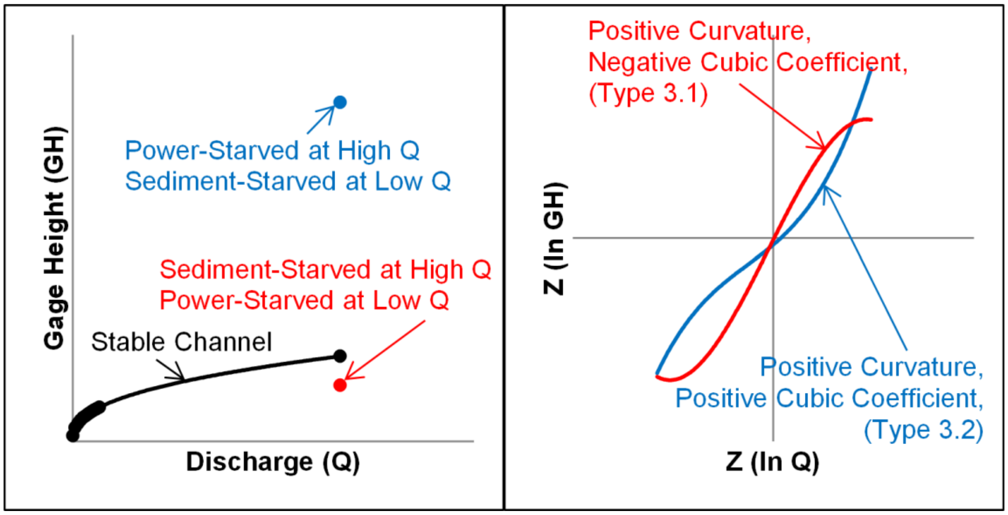

Figure 1b. In a stable channel, a plot of gage height as a function of discharge will be a power-law curve. If scouring (sediment-starvation) occurs at high discharge, deposition (power-starvation) occurs at low discharge, and the gaging site is overall power-starved, then the best-fit parabola will have a positive curvature on a plot of the Z-score of the logarithm of gage height as a function of the Z-score of the logarithm of discharge, while the best-fit cubic polynomial will have a negative cubic coefficient (Type 3.1 rating curve). If deposition (power-starvation) occurs at high discharge, scouring (sediment-starvation) occurs at low discharge, and the gaging site is overall power-starved, then the best-fit parabola will have a positive curvature on the Z-score logarithmic plot, while the best-fit cubic polynomial will have a positive cubic coefficient (Type 3.2 rating curve). Deviations from the power-law curve at low discharge cannot be seen at the scale of the left-hand diagram.

3. Examples

Gaging stations in Utah were chosen randomly out of the USGS National Water Information System (NWIS) database (USGS 2016a) and the set of measurements from the most recent rating number was used to develop a rating curve based on a linear, parabolic or cubic fit on Z-score logarithmic plots. The higher-order polynomial fit was accepted only if the decrease in the goodness-of-fit parameter R^2 from the lower- to the higher-order polynomial fit was statistically significant at the 95% confidence level according to the ANOVA statistical test. Only the first example found for each type of rating curve was examined for consistency with the hydrogeologic criteria (see Fig. 2, Table 1). Tributaries that were unnamed in the National Hydrography Dataset (USGS 2016b) were assumed to have relatively insignificant discharge with small likelihood of contributing reverse flow or flood waves. The stream bank lithology was taken as the mapped unit adjacent to the stream alluvium on a 30'×60' quadrangle map. The stream slope was measured using the USGS 10-meter National Elevation Dataset (USGS 2016c) over an interval centered on the gaging station and extending 50 m upstream and downstream or to the closest tributary if closer than 50 m.

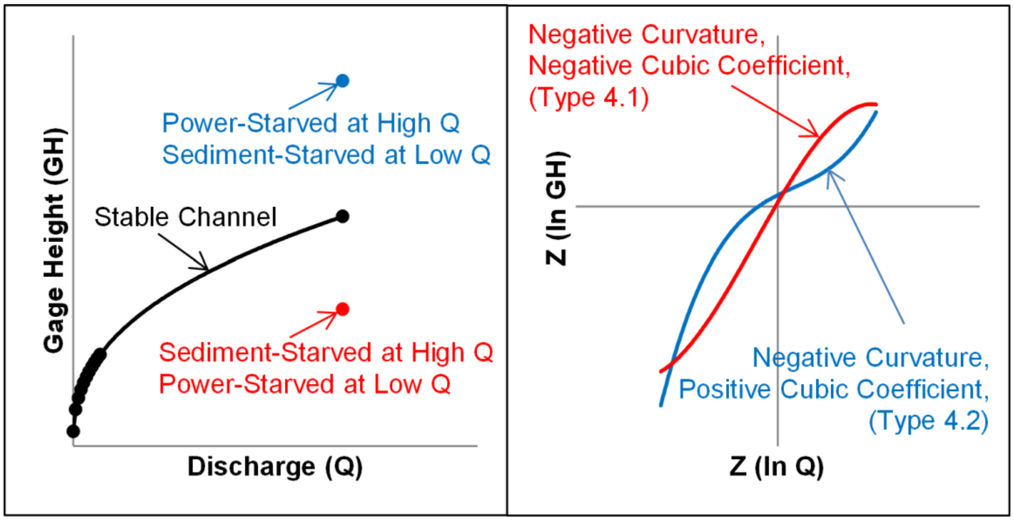

Figure 1c. In a stable channel, a plot of gage height as a function of discharge will be a power-law curve. If scouring (sediment-starvation) occurs at high discharge, deposition (power-starvation) occurs at low discharge, and the gaging site is overall sediment-starved, then the best-fit parabola will have a negative curvature on a plot of the Z-score of the logarithm of gage height as a function of the Z-score of the logarithm of discharge, while the best-fit cubic polynomial will have a negative cubic coefficient (Type 4.1 rating curve). If deposition (power-starvation) occurs at high discharge, scouring (sediment-starvation) occurs at low discharge, and the gaging site is overall sediment-starved, then the best-fit parabola will have a negative curvature on the Z-score logarithmic plot, while the best-fit cubic polynomial will have a positive cubic coefficient (Type 4.2 rating curve). Deviations from the power-law curve at low discharge cannot be seen at the scale of the left-hand diagram.

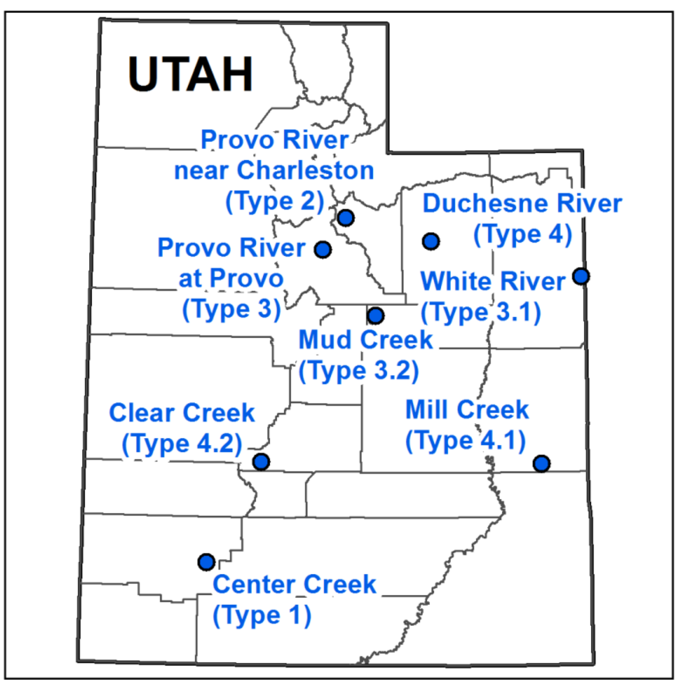

Figure 2. Examples of all eight types of rating curves can be found among the USGS gaging stations in Utah.

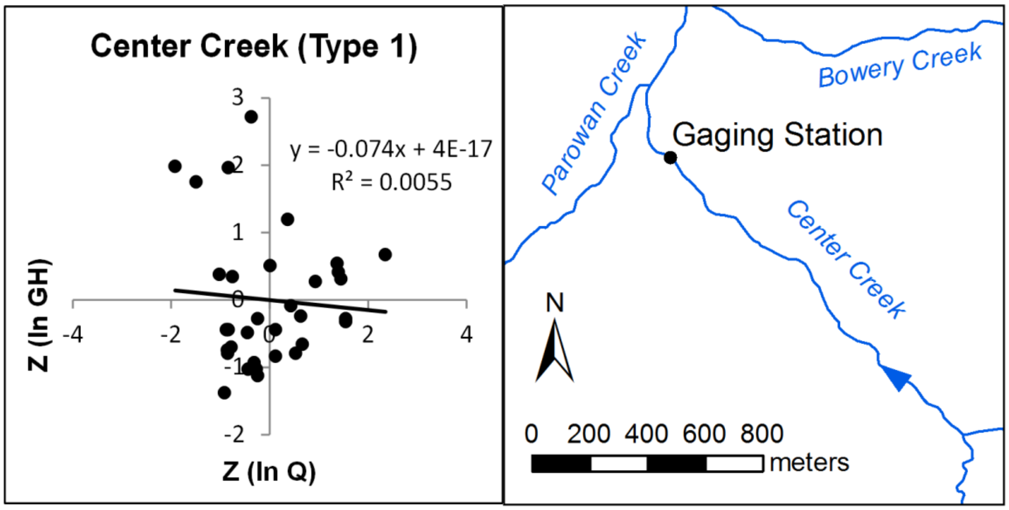

Figure 3a. Rating No. 13 of USGS Gaging Station No. 10241470 (Center Creek above Parowan Creek, near Parowan, Utah) is an example of a Type 1 rating curve (no linear, parabolic or cubic fit). This rating curve is consistent with the proximity of the gaging station to a downstream confluence, which could result in frequent reverse flow.

The eight example rating curves were generally consistent with the hydrogeologic criteria for classifying rating curves (see Figs. 3a-h, Table 1). Only the gaging station close to a downstream confluence with a named stream had a Type 1 rating curve (see Fig. 3a). Although a gaging station with a Type 4 rating curve was also close to a downstream confluence, the adjoining stream was unnamed and could be assumed to have relatively insignificant discharge (see Fig. 3d). The gaging station with a Type 2 rating curve had moderate slope ($S = 0.0075$, see Fig. 3b). Gaging stations with Type 3, 3.1 and 3.2 rating curves had relatively low slope ($S = 0.0029$-0.0053, see Figs. 3c,e,f), while gaging stations with Type 4, 4.1 and 4.2 rating curves had relatively high slope ($S = 0.012$-0.028, see Figs. 3d,g,h). The correspondence between the hydrogeologic criteria for stream bank erodibility and the mapped geology was more straightforward in some cases than others. For example, alkali-rhyolite ash-flow tuff certainly corresponds to the erosion-resistant bedrock stream banks required for a Type 4.2 rating curve (see Fig. 3h). On the other hand, stream banks composed of a mix of terrace deposits, sandstone, shaly siltstone, shale, carbonaceous shale and coal probably have about the same erodibility as the unconsolidated coarse-grained materials required for a Type 3.2 rating curve, although this is not obvious (see Fig. 3f).

4. Discussion

This study has shown that all of the possible hydrogeologic criteria for the synthesis of rating curves can be reduced to the three criteria of (1) stream slope (2) stream bank erodibility (3) distance to the nearest upstream or downstream confluence. The next step will be to create a compilation of the above criteria for a very large number of gaging stations so that a machine-learning algorithm can be developed that will predict the polynomial coefficients of the dimensionless Z-score rating curves, in addition to classifying gaging stations into rating curve types. Once that has been done, only two field measurements of discharge – gage height pairs are required to convert a dimensionless rating curve into a rating curve based on actual discharge and gage height values (see Eqs. 1-2). The major obstacle in machine learning is likely to be the quantification of stream bank erodibility. One possibility is to take a subset of the gaging station database and use the known rating curves as an input to predict a quantitative measure of stream bank erodibility that would be matched with lithologic descriptions. This quantification of stream bank erodibility would then be used as a predictor of rating curves for the rest of the gaging station database and could have application in other kinds of erosion studies.

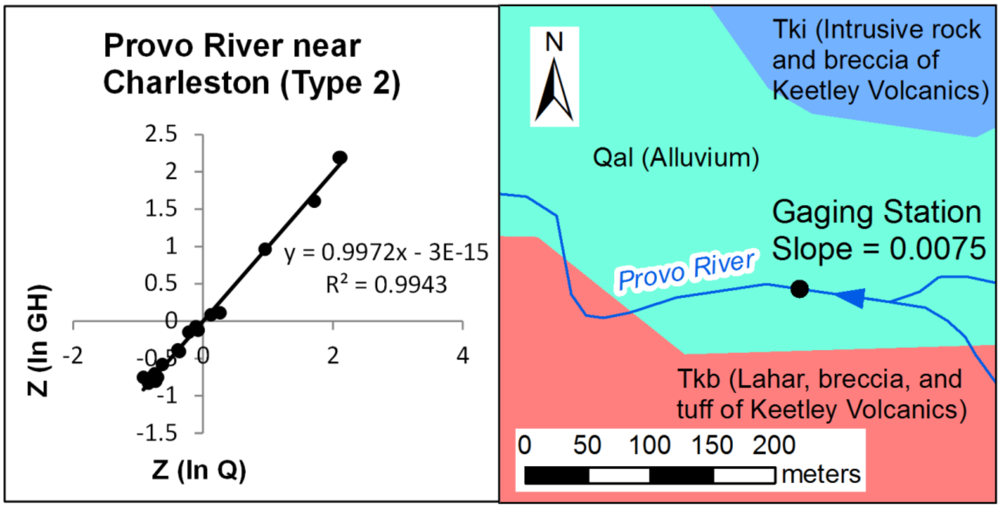

Figure 3b. Rating No. 9.1 of USGS Gaging Station No. 10155500 (Provo River near Charleston River, Utah) is an example of a Type 2 rating curve (parabolic fit no better than linear fit). This rating curve is consistent with a moderate stream slope and the probable moderate erodibility of stream banks composed of a mix of volcanic lahar, breccia and tuff. Geology adapted from the Salt Lake City 30'×60' Quadrangle map (Bryant and Nichols 2003).

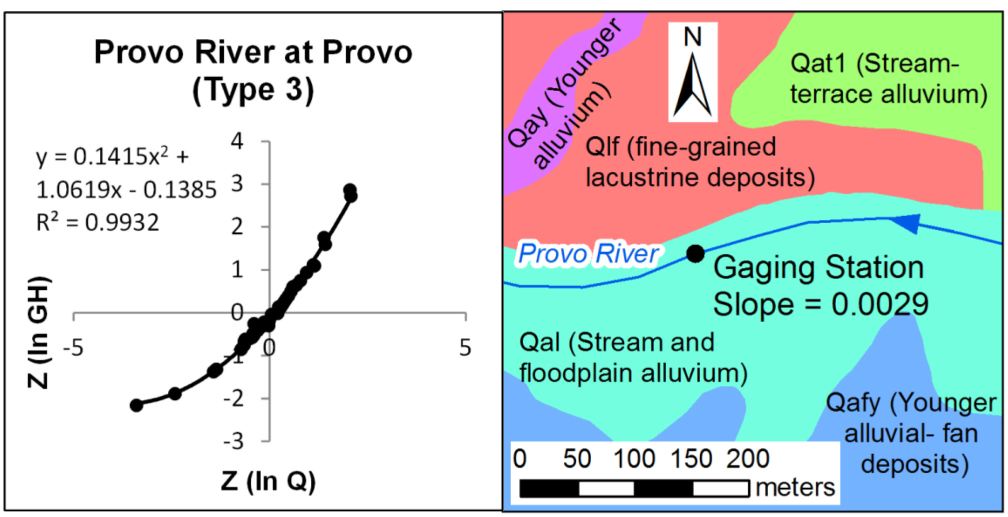

Figure 3c. Rating No. 25 of USGS Gaging Station No. 10163000 (Provo River at Provo) is an example of a Type 3 rating curve (best-fit parabola has positive curvature, cubic fit no better than parabolic fit). This rating curve is consistent with a low stream slope, unconsolidated stream banks and the probable moderate grain size of a mix of alluvial-fan deposits, fine-grained lacustrine deposits, and stream-terrace alluvium. Geology adapted from the Provo 30'×60' Quadrangle map (Constenius et al. 2011).

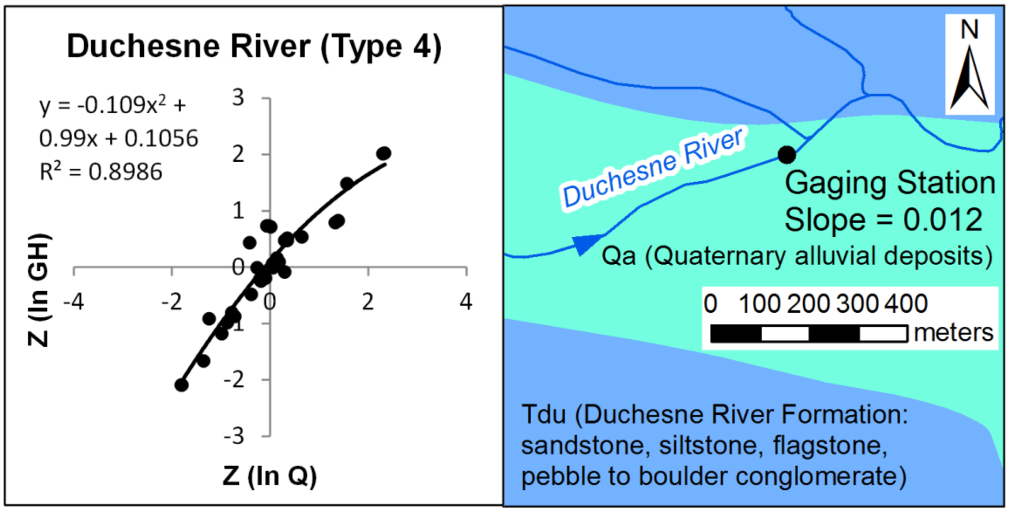

Figure 3d. Rating No. 34 of USGS Gaging Station No. 09277500 (Duchesne River near Tabiona, Utah), is an example of a Type 4 rating curve (best-fit parabola has negative curvature, cubic fit no better than parabolic fit). This rating curve is consistent with a high stream slope and bedrock stream banks that are moderately susceptible to erosion (a mix of sandstone, siltstone, flagstone, and pebble to boulder conglomerate). Unnamed tributaries suggest relatively low discharge and a small likelihood of reverse flow. Geology adapted from the Duchesne 30'×60' Quadrangle map (Sprinkel 2015).

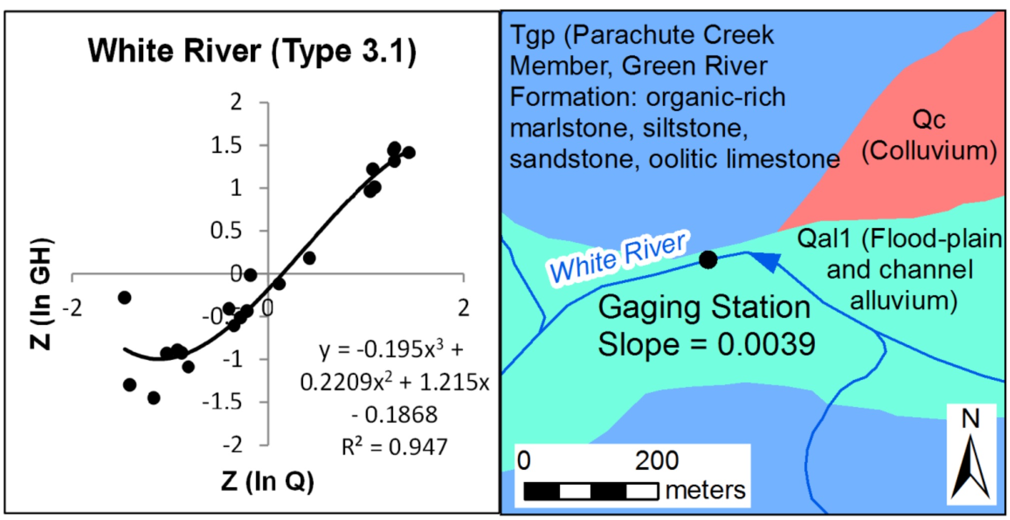

Figure 3e. Rating No. 2 of USGS Gaging Station No. 09306395 (White River near Colorado State Line, Utah) is an example of a Type 3.1 rating curve (best-fit parabola has positive curvature, cubic better than parabolic fit, negative cubic coefficient). This rating curve is consistent with the model of a low stream slope and unconsolidated stream banks with fine grain size, which would probably have the same erodibility as the mix of colluvium, organic-rich marlstone, siltstone, sandstone, and oolitic limestone present in the stream banks at this gaging station. Geology adapted from the Vernal 30'×60' Quadrangle map (Sprinkel 2007).

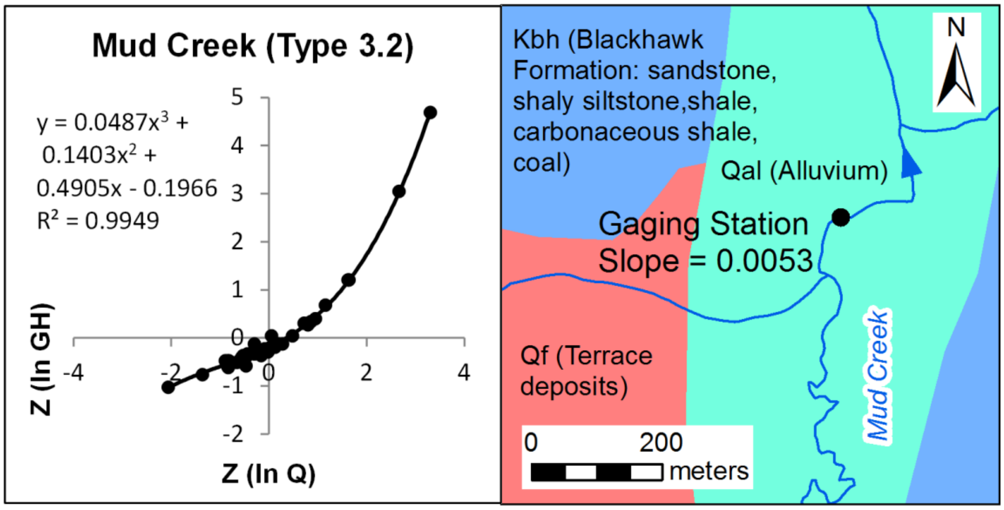

Figure 3f. Rating No. 5 of USGS Gaging Station No. 09310700 (Mud Creek below Winter Quarters Canyon at Scofield, Utah) is an example of a Type 3.2 rating curve (best-fit parabola has positive curvature, cubic better than parabolic fit, positive cubic coefficient). This rating curve is consistent with the model of a low stream slope and unconsolidated stream banks with coarse grain size, which would probably have the same erodibility as the mix of terrace deposits, sandstone, shaly siltstone, shale, carbonaceous shale and coal present in the stream banks at this gaging station. Geology adapted from the Nephi 30'×60' Quadrangle map (Witkind and Weiss, 2002).

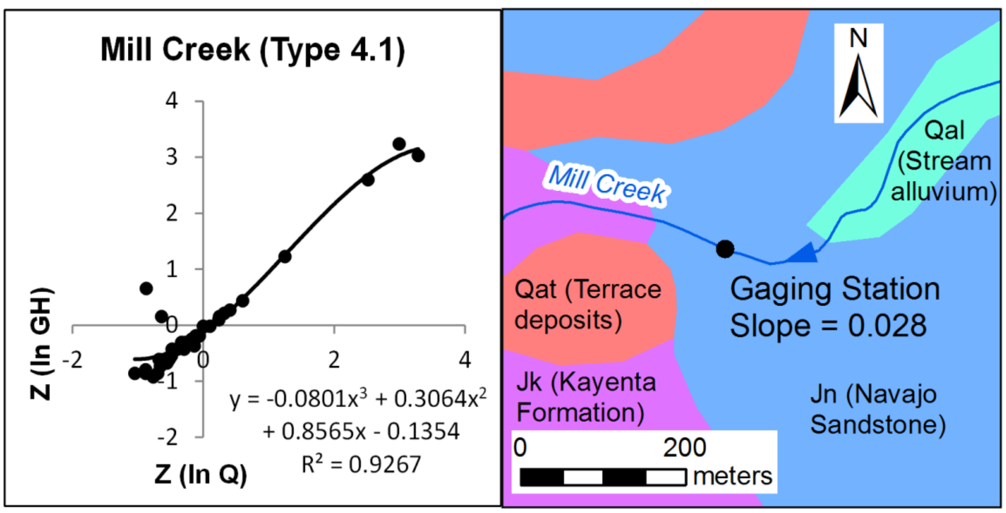

Figure 3g. Rating No. 12 of USGS Gaging Station No. 09184000 (Mill Creek near Moab, Utah) is an example of a Type 4.1 rating curve (best-fit parabola has negative curvature, cubic better than parabolic fit, negative cubic coefficient). This rating curve is consistent with a high stream slope and bedrock stream banks that are relatively susceptible to erosion (sandstone). Geology adapted from the Moab 30'×60' Quadrangle map (Doelling 2002).

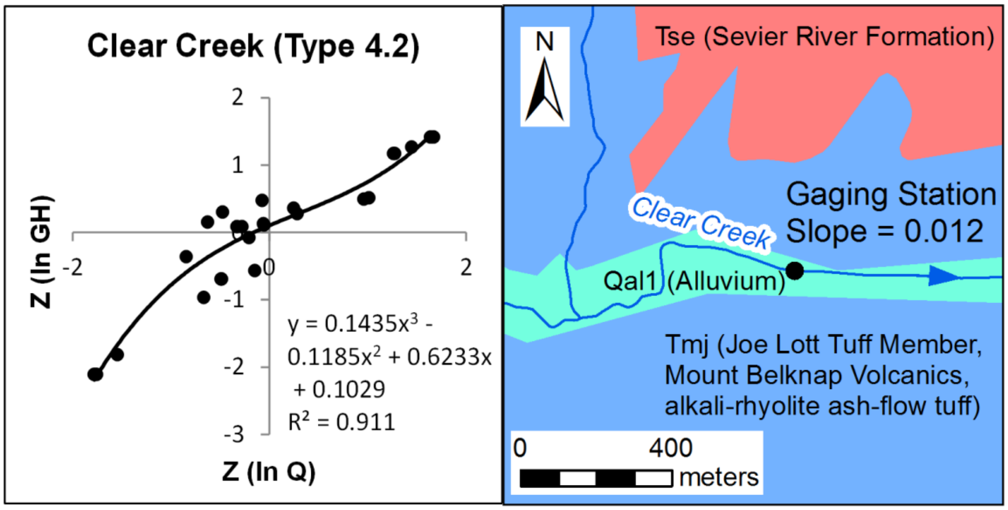

Figure 3h. Rating No. 23 of USGS Gaging Station No. 10194200 (Clear Creek above Diversions, near Sevier, Utah) is an example of a Type 4.2 rating curve (best-fit parabola has negative curvature, cubic better than parabolic fit, positive cubic coefficient). This rating curve is consistent with a high stream slope and bedrock stream banks that are relatively resistant to erosion (alkali-rhyolite ash-flow tuff). Geology adapted from the Richfield 30'×60' Quadrangle map (Hintze et al. 2003).

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