Economic Concept Based Approach for Multiple-Inequality-Constrained Optimization

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Abstract. Many practical optimization problems are multiple-inequality-constrained problems. Optimization to a multiple-inequality-constrained problem is challenging for a complex system. Due to limitation of mathematical programming, direct search optimization approach (stochastic heuristic search methods, such as genetic algorithm, etc.) has been often used to solve real-world problems. However, direct search approach does not evaluate if multiple inequality constraints are approached. Whether optimization is really achieved is often not evaluated.

Multiple-inequality-constrained optimization problem can be evaluated from an economic concept. This concept can be explained using design of an aircraft or design of a levee. Equal strength should be designed for every part; otherwise, when the weakest part breaks, the entire system fails no matter how strong the other parts are. The parts stronger than required can be considered over-performed. Optimization for this type of problem is to strengthen all portions equally and to ensure that every part meets but does not exceed the performance criteria or performance constraints. A well-known proverb says that "a chain is only as strong as its weakest link." This indicates that if multiple inequality performance constraints are not "evenly" approached, optimization has not been achieved yet.

The cost minimization problem can be considered to eliminate the cost from the over-performed portions. To evenly approach the multiple inequality performance constraints, we formed a sequence of dual problems with equality constraints.

The dual problems then can be solved sequentially by Successive Equimarginal Approach (SEA) (Guo, et al, 2007), which was developed based on Equimarginal Principle. An optimal allocation of resources requires that Equimarginal Principle, a fundamental economic principle, must hold; otherwise, resources should be reallocated from producer with lower marginal return to producer with higher marginal return.

For each dual problem, the mathematical logic of the classical gradient descent method is interpreted by the economic concept and is expanded as heuristic rules. According to these heuristic rules, SEA can continuously and flexibly shift resources from producers with lower marginal return to producers with higher marginal return, until (1) marginal returns are almost equal among multiple producers and (2) multiple inequality performance constraints are almost evenly approached. During the process, the search paths can be alternated. Solution can be either feasible or infeasible, back and forth around the inequality performance constraints. Penalty function is avoided, which allows SEA to gain all the feedback from either feasible or infeasible solutions without exaggeration.

SEA has been successfully applied to design a pump-and-treat system for groundwater remediation at Hastings, NE. The optimal design requires that the distributed groundwater contaminant plumes be equally contained without bypass and overprotection be minimized. To achieve that, SEA continuously shifts a pumping rate from a less effective well to a more effective well until equal marginal productivity for all wells is almost reached and the containment boundary is almost evenly touched by the groundwater contaminant plumes in three-dimensions and in time. The application results are significantly improved compared to those using genetic algorithms for the same project as published at the ESTCP website. Details of the SEA method and its application are discussed in Guo, et al (2007) in Water Resources Research.

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