On the turbulent Prandtl in stably stratified turbulence

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Abstract. In this study, we derive a general relationship for the turbulent Prandtl number $Pr_t$ for homogeneous stably stratified turbulence from the turbulent kinetic energy and scalar variance equations. A formulation for the turbulent Prandtl number $Pr_t$ is developed in terms of a mixing lengthscale $L_M$ and an overturning lengthscale $L_E$, the ratio of the mechanical to scalar time scales $T_L/T_\rho$ and the gradient Richardson number $Ri$. We show that our formulation for $Pr_t$ is appropriate even for nonstationary (developing) stratified flows since it does not include the reversible contributions in both the kinetic energy production and buoyancy fluxes that drive the time variations in the flow. Our analysis of direct numerical simulation data of homogeneous sheared turbulence shows that the ratio $L_M/L_E \approx 1$ for weakly stratified flows. We show that in the limit of zero stratification, the turbulent Prandtl number is equal to the inverse of the ratio of the mechanical to scalar time scales, $T_L/T_\rho$. We propose a new parameterization for $Pr_t$ in terms of the gradient Richardson number $Ri$ and use data from stably stratified direct numerical simulations to support it. The formulation presented here provides a general framework for calculating $Pr_t$ that will be useful for turbulence closure schemes in numerical models.

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