

Cellular automata models of particle interactions in sediment entrainment

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Abstract. The geometry of a river bed surface is determined by the arrangement and size of the particles that form the bed. In gravel-to-boulder bed rivers, the surface geometry around any individual particle may be complex and highly variable, and tends to include multi-particle structures. These textural and structural characteristics influence whether an individual particle is entrained. The effect on entrainment is generally described as particle hiding and protrusion, which together define the degree to which an individual particle is sheltered by surrounding particles.

Two variants of a cellular automata (CA) model are used here to investigate the effect of controls on sediment entrainment resulting from particle sheltering. One model variant uses the degree of sheltering as a threshold constraint; the other variant uses the degree of sheltering as an extremal condition. There is a striking contrast in the spatial pattern of particle sizes that result with these different entrainment rules. The extremal rule results in a patchy distribution of particle sizes, in which groups of adjacent cells tend to have the same particle size. The effect of the extremal rule is roughly analogous to considering a sequence of flow events that progressively increase the overall stability of particles on the channel bed, so that over time successively higher flows are required to mobilize particles. In contrast, the threshold rule results in a channel bed that tends to coarsen with time.

1. Introduction

The geometry around individual particles on a river bed surface is determined by the arrangement and size of particles forming the bed. The surface geometry around any individual particle may be complex and highly variable. In steep, rough, gravel-to-boulder bed rivers, the bed surface tends to include features composed of multiple particles. Multi-particle features may include patches of sediment of varying particle size in local areas of the bed

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(e.g., Buffington and Montgomery 1999a), as well as particle clusters and linear to reticulate networks of particles (Church and others 1998). Flow intensity during formation of such textural patterns and particle structures may control the location, form and spacing of these features. The patterns and structures may also be modified by subsequent flows (Lenzi 2001). Textures and structures can also be influenced by random and non-random effects external to the flow, including bank irregularities, rock outcrops and woody debris (e.g., Buffington and Montgomery 1999b). In spite of these complications, a basic representation of textural and structural features on the river bed can be obtained by simply considering them as juxtapositions of particles of different sizes. Such features are expected to be a significant factor in determining whether an individual particle is entrained in steep, rough mountain rivers.

The surface geometry around any given particle can be described in terms of the relative size of adjacent particles. Because the number of possible combinations of particles of different sizes forming a group is very large, even when particle size is subdivided into relatively wide classes, the simplest approach is to use the concepts of particle protrusion into the flow, and particle hiding. Boulders and cobbles hide finer particles from the flow; conversely the boulders and cobbles protrude higher into the flow than the finer sediment. These effects have typically been evaluated in terms of particle size fractions relative to a characteristic particle size such as the median, D_{50} . Hiding effects then refer to effects on particles smaller than the median; protrusion refers to effects on particles larger than the median. Here, the geometric effects of size differences among adjacent particles are considered relative to individual particles rather than relative to a characteristic particle size. To avoid confusion with the frequently used sense of hiding and protrusion, these effects are referred to as particle sheltering.

Cellular automata (CA) are models in which time, space and all variables take on only discrete values, and the model response in each cell is determined by the current state of the cell and of a set of neighboring cells (Hergarten 2002, p. 136). A CA model to investigate some effects on sediment entrainment resulting from particle sheltering is presented here. This cellular automaton allows an initial investigation of entrainment effects resulting from particle sheltering. As used here, the CA is a tool for isolating a single effect and evaluating its consequences.

In the model presented here, the states that each cell in the model may assume are defined as different degrees of particle sheltering. The effect of particle sheltering may operate in several different ways. Two model variants have been developed using different characterizations the effects of particle sheltering. One model variant uses the degree of sheltering as a threshold constraint; the other variant uses the degree of sheltering as an extremal condition.

The threshold model variant uses the approach that, for a given flow, there is some minimum height of particle exposure necessary to allow entrainment. This fixed threshold approach is most likely to approximate

conditions in a system with steady flow, and thus may be applicable for minutes or hours during flood flows, and perhaps days or weeks at low flows.

The model variant using an extremal rule is based on the expectation that only unstable particles will move at any given time. Whether a grain is unstable may vary with time, e.g., with changing flow. For longer-term evolution, it has been hypothesized that the particulate surface may evolve toward a condition that maximizes resistance to flow and the particles at the surface may develop a maximal resistance to entrainment (Davies and Sutherland 1980, 1983). Such extremal hypotheses have a long history in physics (Auyang 1998, p. 80, 83), and it has recently been suggested (Bak 1996, p. 103) that extremal rules may be necessary for the development of self-organized criticality. The CA using an extremal rule presented here has some similarities to the quintessential SOC system of a model sandpile, involving movement of grains down the sandpile slope. However, there are also significant differences when the system is a gravel-to-boulder bed stream, including limits to the channel width, constituting the system size, relative to the smallest event size, given by movement of a single particle. In addition, initiation of particle motion on the stream bed occurs by processes very different than the process of avalanching on a model sandpile.

2. Model and variants

Particle sheltering is defined in the model as the height of a selected particle or particles covering some unit area of the channel bed, relative to the average height of the surrounding particles. The model variants are totalistic CA because the model process depends on the sum of the states in neighboring cells, rather than on a specific spatial arrangement of states in the neighborhood, or individual states in the neighboring cells. In the model variants, the surrounding particles considered to contribute to particle sheltering are limited to the upstream, downstream, and left and right lateral particles.

The particles and the entrainment process are evaluated on a scale no finer than the cell dimension. Each cell contains one or more particles of a given size. The initial size in each cell is chosen randomly from a set of particle sizes that are one phi size apart. In each of the runs presented here, the largest particle is arbitrarily selected as 256 mm, and the remaining particle classes are 128 mm, 64 mm, etc. Model grids presented here have 5 size classes.

All height differences are measured relative to a single plane at level zero. The height difference between the particles in the selected cell and the average height of particles in surrounding cells is termed the shelter margin. The shelter margin is positive for particles that, on average, rise above the surrounding particles, and negative for particles that, on average, do not rise above the surrounding particles.

Two alternative approaches are used to specify boundary conditions. In one approach, cyclic boundaries are used on all sides of the model. In the other approach, fixed lateral boundaries are used with cyclic up/downstream

boundaries. The model with all boundaries defined as cyclic boundaries can be thought of as representing conditions near the center of a wide, long channel, either a physical channel or in the model. Particle output at the downstream boundary of the reach is assumed to approximate what 'would have been' transported or modeled as inputs at the top or lateral margins of the reach. The approach with fixed lateral boundaries can be interpreted as analogous to non- or weakly-erodible banks or flume walls.

Each model was run with each set of boundary conditions. In runs with fixed lateral boundaries, the boundaries have an influence on particle size patterns within a few cell distances from the boundaries. However, there is little difference in the particle-size patterns that form in the model channel as a whole. The examples presented here represent only runs with fully cyclic boundary conditions. Model grids presented are 15 cells wide by 80 cells long.

2.1. Model variant with an extremal rule

In the model variant using the 'least stable particles move' extremal rule, only those particles in cells that are the least sheltered are mobile in any time step. The least sheltered particles are those that have a shelter margin equal to the maximum shelter margin in the current grid.

Using this rule, an initially random pattern of particle sizes evolves until the cells of one particle size are more frequently adjacent to cells with

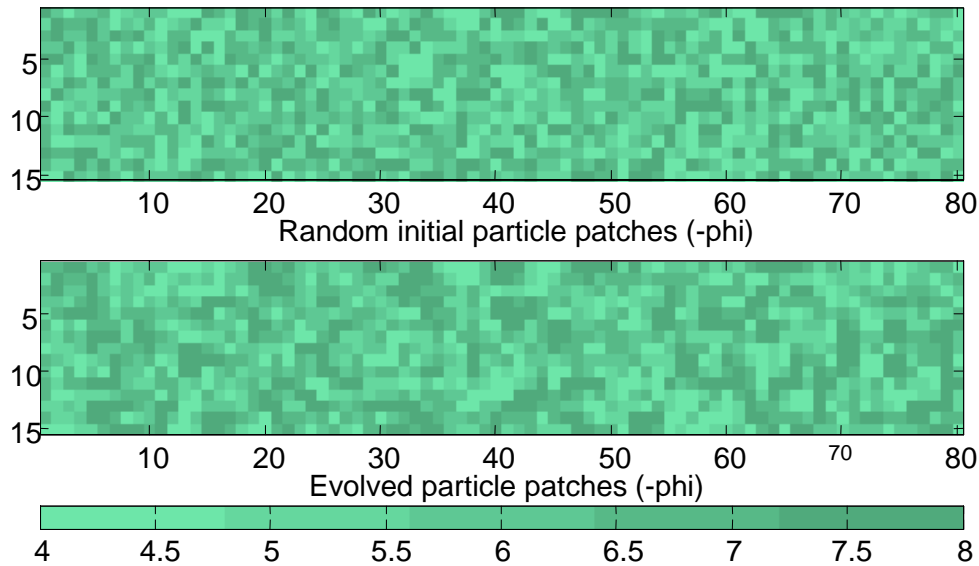


Figure 1. Initial and final grids using the 'least stable moves' extremal rule. Model run for 300 time steps using cyclic boundary conditions.

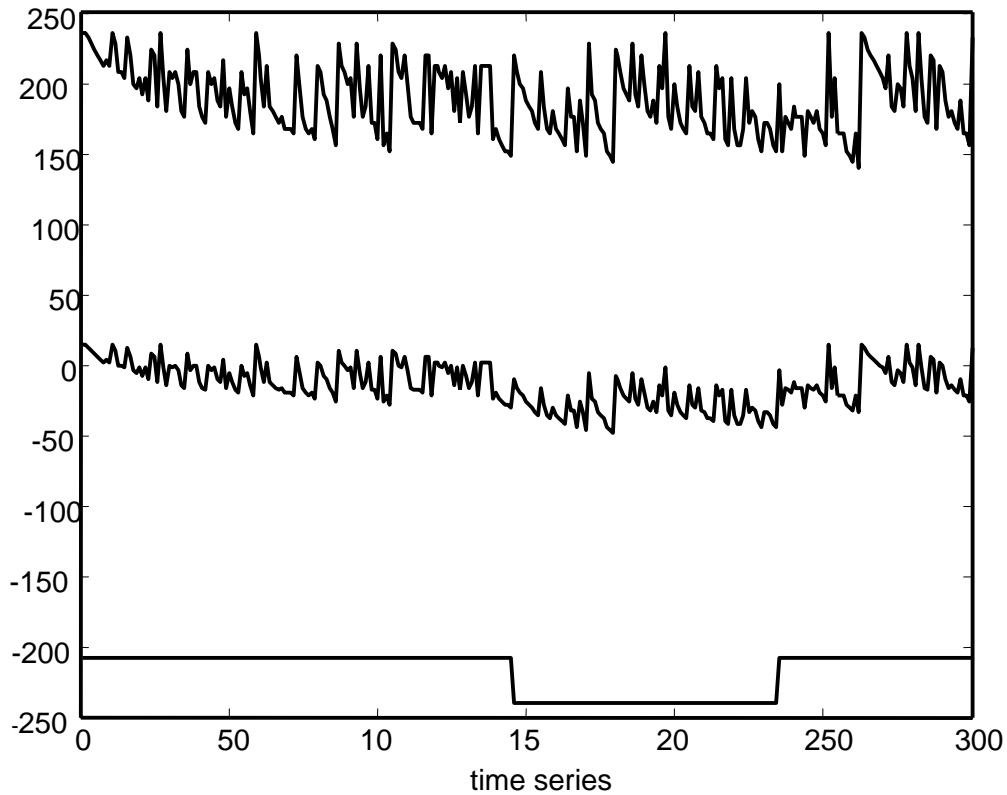


Figure 2. Time series showing evolution of the shelter margin with the 'least stable moves' extremal rule. The upper curve gives the maximum shelter margin in the grid at each time step; the lower curve gives the minimum; the middle curve is the average of the maximum and minimum. Model run for 300 time steps using cyclic boundary conditions.

the same particle size, forming patches of large particles and patches of small particles. The patterns may best be described as patches with reticulate connections. An example grid showing the initial state, as well as the patches formed after 300 time steps, is shown in Figure 1.

As the grid evolves from its original, random distribution of particle sizes, there is an initial rapid drop in the maximum shelter margin, shown in Figure 2, reflecting motion of the least sheltered particles. A brief, continuous decrease in the maximum shelter margin continues for roughly 10 to 20 time steps in model runs. The maximum shelter margin continues to decrease overall for about 75 to 100 time steps, but with significant fluctuations within cycles of higher and lower shelter margin. Longer runs suggest that the frequency of these cycles may decrease with time, while the amplitude of the cycles may increase.

2.2. Model variant with a threshold rule

The second model variant uses a single, fixed threshold value of the shelter margin, and defines particles as mobile if the shelter margin exceeds the threshold. The rule can be summarized by saying a particle is 'mobile if

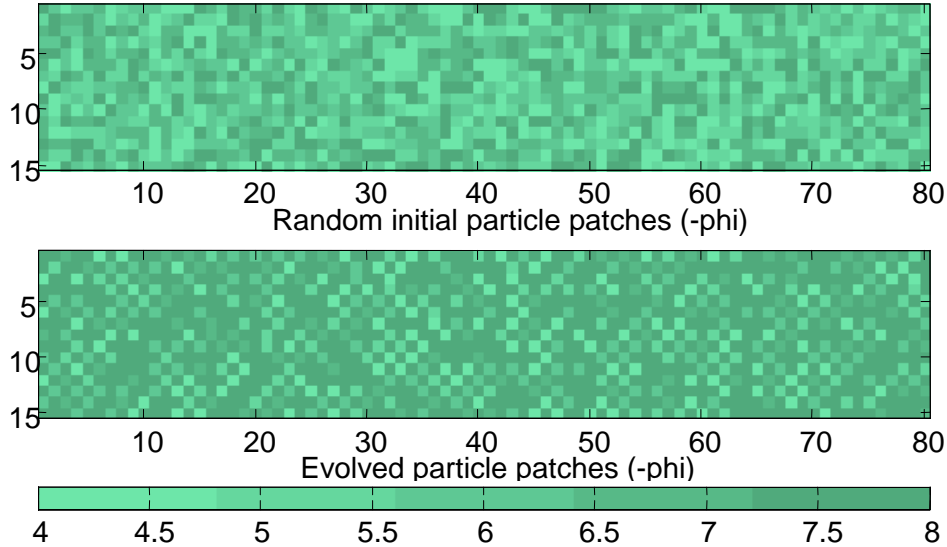


Figure 3. Threshold model variant with particle mobility if the shelter margin is below zero. Model run for 300 time steps using cyclic boundary conditions.

exposed.' In the model run illustrated in Figures 3 and 4, the threshold was set to a shelter margin of zero, i.e., particles in a cell move only if the difference between the particles' height and the average height of the neighborhood particles is positive. With this rule, the model grid coarsens with time. The checkerboard pattern within the predominantly coarse surface shown in Figure 3 is an artifact of the shelter margin definition dependence on only the lateral and up/downstream particles.

With this threshold rule, and given the initial random spatial distribution of particle sizes, the larger particles initially tend to be less sheltered and more mobile than small particles. However, as large particles move, they tend to remain longer at locations where they are adjacent to other large particles, because they are more often sufficiently sheltered at these locations. A large particle with other large particles on all sides is temporarily immobile until one of the adjacent large particles moves. Thus, large particles are highly mobile in the random initial grid, and become essentially immobile over time as the surface coarsens.

An example of the shelter margin evolution using the threshold rule is shown in Figure 4. With this rule, maximum and minimum values of the shelter margin vary only slightly with time. There is an initial, relatively small drop in the shelter margin, in which the average of the minimum and maximum shelter margins goes from positive to negative, then rises to the threshold value. After this initial adjustment, the shelter margin fluctuates frequently over time, but the fluctuations are small.

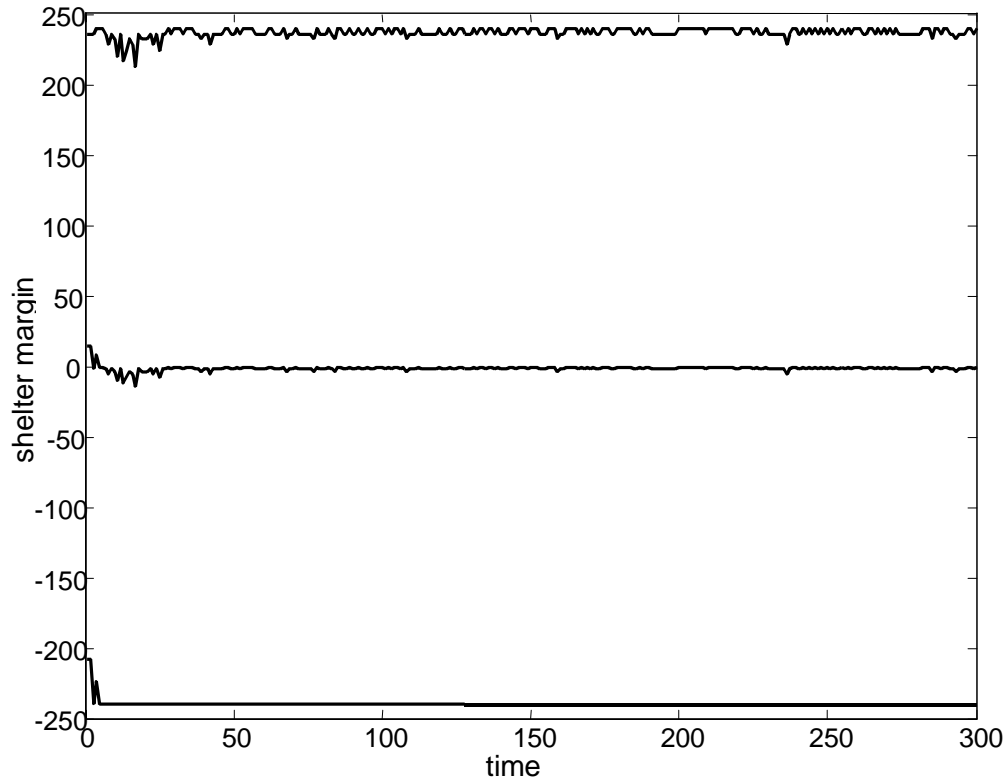


Figure 4. Time series showing evolution of the shelter margin (in mm) for the 'mobile if exposed' threshold rule. Here the threshold was set to a shelter margin of zero. The upper curve gives the maximum shelter margin in the grid at each time step; the lower curve gives the minimum; the middle curve is the average of the maximum and minimum. Model run for 300 time steps using cyclic boundary conditions.

3. Results and discussion

There is a striking contrast in the spatial pattern of particle sizes that result with different forms of an entrainment rule based on the degree of particle sheltering. The 'least stable moves' extremal rule results in a patchy distribution of particles sizes, where groups of adjacent cells tend to have the same particle size. Particle patches are roughly equidimensional, and each patches occupies only a fraction of the channel width after several hundred time steps in the model grids used here. The patches of coarse particles are frequently connected by lines of large particles, reminiscent of the reticulate networks described by Church and others (1998).

An interpretation of the model evolution under the extremal rule is that successive time steps represent a sequence of flow events just sufficient

to entrain the least stable particles on the bed, thus corresponding to sediment transport events. Over time, there is a tendency for successively higher flow to be required for particle entrainment in successive events. Using this interpretation, each time step in the model may represent one or more years, and the model runs presented here represent relatively long-term evolution of the system over hundreds of years. The extremal rule has some similarity to the extremal hypothesis of maximum particle stability proposed by Davies and Sutherland (1980, 1983). However, although the evolutionary pattern in the model is controlled by removal of particles from unstable positions on the bed surface, the next position of an entrained particle may or may not be more stable than its previous location. Stability at the next site depends instead on the local bed geometry at the new site. Rather than evolving to maximum stability overall, particle stability using the least stable moves extremal rule fluctuates significantly over time. Based on an initial review of model results, these fluctuations appear to develop a longer cycle time as the system evolves; the cycle amplitude may also increase.

The formation of sediment patches when the extremal rule is used can also be described as a self-organization of the channel bed. Although the channel bed appears to be self-organized, additional work is needed to determine whether it stabilizes at a self-organized critical (SOC) state.

In the model variant using a fixed threshold of the shelter margin as a criterion for particle entrainment, the surface of the model channel essentially coarsens over time. Coarsening of the model bed surface is a consequence of high mobility of isolated coarse particles and low mobility of clustered coarse particles.

The evolution of the shelter margin also differs between the 'least stable moves' extremal model variant and the threshold variant. With the extremal rule, there is initially a brief, rapid drop in the value of the minimum shelter margin. In these first few time steps, a set of very unstable grains are entrained, and progressively moved to new positions that tend to have greater stability, indicated by the drop in the shelter margin. The maximum shelter margin then continues to slowly decrease on average, representing decreasing roughness on average. Except for the brief initial decrease, the maximum shelter margin fluctuates widely as it decreases on average. A tendency for the minimum shelter margin to increase with time is seen in some runs. After the initial decrease in surface roughness, the shelter margin using the extremal rule undergoes significant oscillations, and rises and falls in cycles.

In contrast, evolution of the shelter margin under the threshold rule results in smaller changes in the minimum and maximum shelter margins over time. With this rule, fluctuations in the average of the minimum and maximum shelter margin oscillate about the threshold value of the shelter margin.

4. Conclusions

The cellular automata model presented here illustrates the consequences of two different ways that the simple effect of the sheltering of

smaller particles by larger particle may affect particle arrangement on the channel bed surface. First, over time periods of minutes to hours, during which flow depth and velocity are relatively steady, a fixed height of sheltering may constitute a threshold for particle entrainment. A threshold model variant using a fixed value of the shelter margin results in a channel surface that coarsens with time.

A different approach was taken in the second model variant, where an extremal hypothesis is assumed. A tendency of the channel bed to become more stable with time is likely to result from movement of unstable particles, but no mechanism ensures that the particles will be more stable on deposition. A rule incorporating such entrainment of the least stable particles results in a channel bed composed of patches with small particles and patches with large particles. Large-particle patches tend to be connected by lines of large particles. As implemented, the rule effectively ignores flows that do not move particles, and each time step may represent a sequence of successively higher flow maxima, separated by a year or more in time.

Assuming that the value of the shelter margin can be equated with particle stability, the pattern of evolution with the extremal rule is a constantly fluctuating degree of particle stability, while simultaneously evolving toward a greater average stability, at least over several hundred time steps. The self-organization of the modeled channel surface into patches of finer and coarser sediment suggests that the model may have the characteristics of a self-organized critical system. Additional work is planned to evaluate that possibility, as well as the influence of additional factors not included in the initial model and its variants.

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