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Improving MODFLOW's RIVER Package for Unsaturated Stream/Aquifer Flow

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Abstract. Saturated flow is typically assumed for seepage from a stream underlain by an alluvial aquifer. However, if the water table in an unconfined aquifer falls a sufficient distance below the streambed, the head losses in this less conductive layer will cause the region beneath the streambed to become unsaturated. Unsaturated flow transforms streams from constant head boundaries to constant flux boundaries, impacting not only the quantity of stream recharge but also biogeochemical transformations. The objective of this paper is to analyze the impact of unsaturated flow on stream/aquifer exchange. The modeling capabilities of one of the most commonly used groundwater flow models, MODFLOW, in simulating unsaturated stream/aquifer exchange is improved. The effects of unsaturated flow on MODFLOW predictions of aquifer drawdown and stream leakage are illustrated for hypothetical stream/aquifer systems.

1. Introduction

MODFLOW is a widely used, finite-difference flow model for simulating saturated groundwater flow and is capable of simulating the interaction between streams and underlying alluvial aquifers. MODFLOW models stream/aquifer interaction using the RIVER or STREAM packages (Harbaugh and McDonald, 1996). The RIVER package assumes that the stream stage remains constant throughout a given stress period within the model. This constant stream stage is then utilized to calculate the flux of water between the stream and aquifer system, proportional to the head gradient between the river and aquifer and a streambed conductance parameter. Limitations in modeling the stream with a constant head in a given stress period led to the development of the STREAM package. The STREAM package is a streamflow routing model limited to steady flow through a rectangular, prismatic channel (Prudic, 1989).

Researchers are stressing the importance of unsaturated flow between a stream and underlying alluvial aquifer (Fox and Durnford, 2003; Osman and Bruen, 2002; Rushton, 1999). MODFLOW's RIVER and STREAM packages appropriately account for saturated flow but account for unsaturated flow by making simplifying assumptions. When the aquifer head is above the bottom of the streambed (i.e., saturated flow), MODFLOW assumes that the specific discharge through the streambed is proportional to the difference in hydraulic head between the stream and aquifer:

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$$q = \frac{K_{sb}}{M} s_w \quad (1)$$

where q is the specific discharge [LT^{-1}] with a downward flux assumed positive, K_{sb} is the streambed hydraulic conductivity [LT^{-1}], M is the streambed thickness [L], and s_w is the drawdown defined as the difference between the hydraulic head in the stream, h_w and the hydraulic head in the aquifer, h. If the aquifer head drops below the bottom of the streambed, MODFLOW's packages assume that the seepage is no longer proportional to the aquifer head and becomes dependent on the water level in the stream and the streambed thickness, M:

$$q = \frac{K_{sb}}{M}\left(H_w + M\right) \quad (2)$$

where H_w is the water level in the stream above the surface of the streambed.

The objective of this paper is to review research on the impact of unsaturated flow on stream/aquifer exchange and improve the modeling capabilities of MODFLOW. This paper will demonstrate that assuming the seepage flux from the stream to be dependent on the elevation of the bottom of the streambed is an inaccurate simplifying assumption.

2. Saturated/Unsaturated, Stream/Aquifer Exchange

Fox and Durnford (2003) provide a detailed analysis of the impact of unsaturated flow on stream/aquifer exchange. A summary of the analysis by Fox and Durnford (2003) is provided in the following paragraphs. A homogeneous and isotropic streambed is assumed to exist between the stream and underlying aquifer, as shown in Figure 1. The streambed is assumed to have a lower hydraulic conductivity than the underlying soil (Calver, 2002). It is also assumed that leakage from the stream to the aquifer is steady, vertical, one-dimensional flow. Fox and Durnford (2003) identified three stream/aquifer hydrologic states or regimes:

- Regime A – Saturated Flow: Stream leakage rate depends on the location of the water table.
- Regime B – Transition zone: Stream leakage rate governed by unsaturated flow hydraulic conditions.
- Regime C – Unsaturated Gravity-Driven Flow: Stream leakage rate is not a function of the location of the water table.

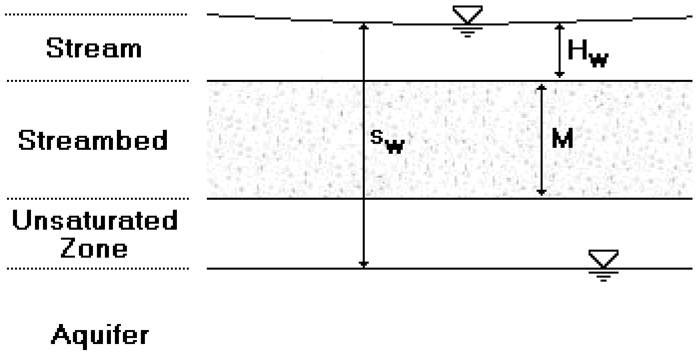

Figure 1. Representation of interaction between a perched stream and underlying alluvial aquifer. H_w=river stage, s_w=drawdown, and M=streambed thickness.

2.1 Regime A: Saturated Flow

Saturated flow occurs in the stream/aquifer interface when the water table is located within the streambed, when the water table is slightly below the bottom of the streambed but water pressures are not sufficiently negative to desaturate the subsoil, or if there are no pathways for the air phase to enter the hyporheic zone. When the water table is located at an elevation corresponding to the bottom of the streambed, the specific discharge, q, through the streambed (assuming one-dimensional, vertical flow, where a downward flux is assumed to be positive) is:

$$q = K_{sb} \frac{H_w + M}{M} \tag{3}$$

where K_{sb} is the saturated hydraulic conductivity of the streambed, H_w is the water level in the stream above the streambed, and M is the streambed thickness. If the water table declines to an elevation below the bottom of the streambed, the water pressure head, h_w, at the bottom of the streambed becomes negative. The specific discharge is then given by:

$$q = K_{sb} \frac{H_w + M + h_c}{M} \tag{4}$$

where h_c is the capillary pressure head at the bottom of the streambed in which h_c is assumed to be equal to the negative of h_w.

The stream/aquifer interface will remain saturated if the capillary pressure head is less than an air entry capillary pressure head, h_e. The air entry pressure head is that capillary pressure head below which the nonwetting phase (in this case air) becomes discontinuous and is not capable of flowing through the material (Corey, 1994). The entry pressure head loosely represents the height of the capillary fringe in a uniform soil. Also, if there are no pathways for air to enter the zone separating the streambed and aquifer, the region below the streambed will also remain saturated no matter how far the water table drops below the streambed. In this case, water pressure heads could theoretically become very large negative values. This paper will assume that pathways exist for air to enter the zone beneath the streambed.

2.2 Regime B: Transition Flow

Unsaturated flow occurs in the zone beneath the streambed when the water table falls a sufficient distance so that some of the pores desaturate. The specific discharge of water through this unsaturated zone is given by:

$$q = K\left(h_c\right)\left(1 - \frac{\partial h_c}{\partial z}\right) \qquad (5)$$

where h_c is the capillary pressure head, $K(h_c)$ is a constitutive relation between hydraulic conductivity and capillary pressure, and z is the depth within the hyporheic zone. The Brooks-Corey equations are commonly used to represent $K(h_c)$:

$$\begin{aligned} K\left(h_c\right) &= K_s & \qquad h_c &\le h_e \\ K\left(h_c\right) &= K_s\left(\frac{h_e}{h_c}\right)^{\eta} & \qquad h_c &> h_e \end{aligned} \qquad (6)$$

where K_s is the saturated hydraulic conductivity, h_c is capillary pressure head, h_e is the entry capillary pressure head, and η is a parameter dependent on the pore-size distribution index (Corey, 1994). Rearranging equation (5) and replacing $K(h_c)$ with the Brooks-Corey equation results in:

$$\frac{\partial h_c}{\partial z} = 1 - \frac{q}{K_s}\left(\frac{h_c}{h_e}\right)^{\eta} \qquad h_c > h_e \qquad (7)$$

where $\partial h_c/\partial z$ is the capillary pressure head gradient with elevation in the unsaturated zone.

The specific discharge in regime B has a magnitude between those in regimes A and C, but under most circumstances, the length of time that this regime applies is small. Using representative values of Brooks-Corey parameters for different soil types, Fox and Durnford (2003) show that the capillary pressure gradient $\partial h_c/\partial z$ will go to zero quickly, especially for medium and coarse sand subsoils. When $\partial h_c/\partial z=0$, a unit hydraulic gradient exists in the unsaturated zone. The formation of this unit hydraulic gradient signals the end of regime B.

2.3 Regime C: Unsaturated Gravity-Driven Unsaturated Flow

Regime C is defined by the presence of only gravity-driven flow ($\partial h_c/\partial z=0$) in contrast to the other regimes where there is also a pressure gradient in the unsaturated zone beneath the streambed. In regime C, stream leakage reaches a maximum as the capillary pressure head at the bottom of the streambed, h_c, becomes a maximum. This maximum interface capillary pressure head will be referred to as the ultimate interface capillary pressure head, h_{cu}, and is the result of the capillary pressure head gradient with elevation becoming zero. The specific discharge through the streambed and unsaturated zone, q_{max}, is given by the following equations, respectively:

$$q_{\max} = K_{sb}\left[1+\frac{(h_{cu}+H_w)}{M}\right] \tag{8}$$

$$q_{\max} = K_s\left(\frac{h_e}{h_{cu}}\right)^{\eta} \qquad h_{cu} > h_e \tag{9}$$

Equations (8) and (9) can be solved simultaneously for q_{max} and h_{cu}.

Figure 2 shows an example from Fox and Durnford (2003) assuming a stream with flow depth, H_w=0.5 m, a streambed thickness, M=1 m, and hydraulic conductivity, K_{sb}, of 0.1 m/day, overlays a subsoil with a saturated hydraulic conductivity, K_s, of 10 m/day, η=6.5, and h_e=0.1 m. These values are characteristic of a silt streambed overlying a medium, sandy aquifer (Carsel and Parrish, 1988). Figure 2 plots the interface capillary pressure head, nondimensionalized by the entry pressure head, as a function of position of the water table below the streambed, nondimensionalized by the streambed thickness, M. During saturated flow (regime A), pore spaces are completely filled with water because the water pressure heads are not sufficiently negative to desaturate the pores. The interface capillary pressure head increases with increasing drawdown in regime B, reaching a constant, maximum capillary pressure head. Once h_{cu} is reached, the capillary pressure head does not increase with additional declines in the water table. A corresponding figure for specific discharge through the streambed can be developed that shows (a) stream leakage varies with water table position in regimes A and B, (b) the transition from regime A to regime C occurs over a small range of drawdown, (c) a constant maximum leakage rate, $|q_{max}|$, is obtained when a unit hydraulic gradient occurs in regime C, and (d) the stream leakage does not change with any additional decline in water table position in regime C.

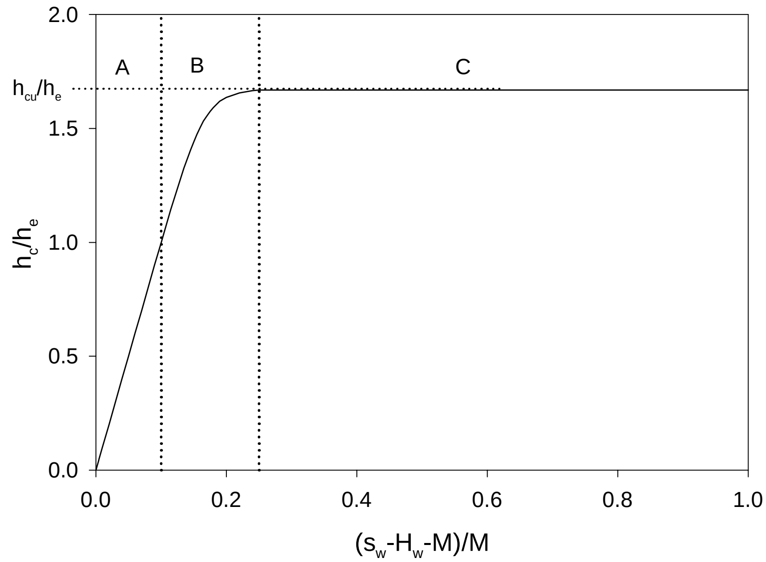

Figure 2. Interface capillary pressure head, h_c, divided by the entry pressure head, h_e, as a functions of water table position for regimes A (saturated flow), B (transition), and C (unsaturated flow). From Fox and Durnford (2003).

3. Modifications to MODFLOW's RIVER Package

The current version of the RIVER package accounts for saturated flow (regime A) and uses a simplifying assumption for unsaturated flow (regime C) as shown in equation (2). Osman and Bruen (2002) presented an improved method for incorporating unsaturated flow into the RIVER package. Their research made use of the van Genuchten (1980) relationship for the capillary pressure head-hydraulic conductivity relation. Additionally, Osman and Bruen make the simplifying assumption that the seepage during the transition regime (regime B) is proportional to the difference in the water level in the stream and the aquifer head. This assumption is equivalent to assuming that flow conditions during regime A hold during regime B.

The proposed improvement to the RIVER package within MODFLOW is based on the use of the Brooks-Corey (1964) relationship. Similar to the current version of the RIVER package, saturated flow is assumed to occur as long as the hydraulic head in the aquifer is above the bottom of the streambed (i.e., $s_w \leq H_w + M$). Seepage between the stream and aquifer is governed by equation (1). Saturated flow also occurs as long as the capillary pressure head does not exceed the air entry pressure head (i.e., $s_w \leq H_w + M + h_e$).

Regime C occurs when the aquifer head falls a sufficient distance below the bottom of the streambed such that a maximum capillary pressure head, h_{cu}, will form at the bottom of the streambed. The maximum capillary pressure head is given by

$$h_{cu} = h_e \left(\frac{K_s}{|q|} \right)^{1/\eta} \tag{10}$$

Specific discharge through the streambed and unsaturated zone will be governed by equations (8) and (9), which can be solved simultaneously for q_{max} and h_{cu}.

The transition regime occurs when the hydraulic head is sufficiently far below the bottom of the streambed (i.e., $s_w > H_w + M + h_e$) but not far enough below to create a unit hydraulic gradient (i.e., $s_w < H_w + M + h_{cu}$). A capillary pressure head is created at the bottom of the streambed that increases the force to induce water through the streambed, such that seepage through the streambed is given by:

$$q = \frac{K_{sb}}{M}\left(H_w + M + h_{cl}\right) \tag{11}$$

where h_{cl} is the capillary pressure head at the bottom of the streambed. Seepage through the unsaturated zone during regime B is given by equation (5). Equation (5) can be rearranged in the form of equation (7) and then integrated from the water table to the bottom of the streambed to relate the drawdown, s_w, and the capillary pressure head at the bottom of the streambed, h_{cl}, in terms of the hydraulic conductivity of the subsoil and soil parameters, h_e and η:

$$s_w - M - H_w = \int_0^{h_{cl}} \frac{\partial h_c}{1 - \frac{q}{K_s}\left(\frac{h_c}{h_e}\right)^{\eta}} \tag{12}$$

The integral in equation (12) can be expressed as a Lerch Phi function (Erdelyi, 1953):

$$s_w - M - H_w = \frac{h_{cl}}{\eta} LerchPhi\left(\frac{q}{K_s}\left(\frac{h_{cl}}{h_e}\right)^{\eta}, 1, \frac{1}{\eta} \right) \tag{13}$$

where LerchPhi(z,a,v) is given by the following infinite series as long as $|z|<1$ (Erdelyi, 1953):

$$LerchPhi(z,a,v) = \sum_{n=0}^{\infty} \frac{z^n}{(v+n)^a} \tag{14}$$

Equations (11) and (13) can be solved simultaneously for q and h_{cl}.

Cases can exist when the transition regime represents a significant component of the hydrologic exchange between a stream and aquifer. Regime B becomes more significant as the difference between the entry capillary pressure head, h_e, and the ultimate capillary pressure head, h_{cu}, increases. Equations (8) and (9) are combined to express h_{cu} as a function of K_{sb}/K (degree of restriction at the streambed), H_w/M (driving force through streambed), h_e, and η. Absolute differences between h_{cu} and h_e for coarse, medium, and fine sand subsoil beneath the streambed are shown in Figure 3. Regime B can be assumed negligible for coarse sand aquifers even with considerable streambed restriction.

(a) (b)

(c)

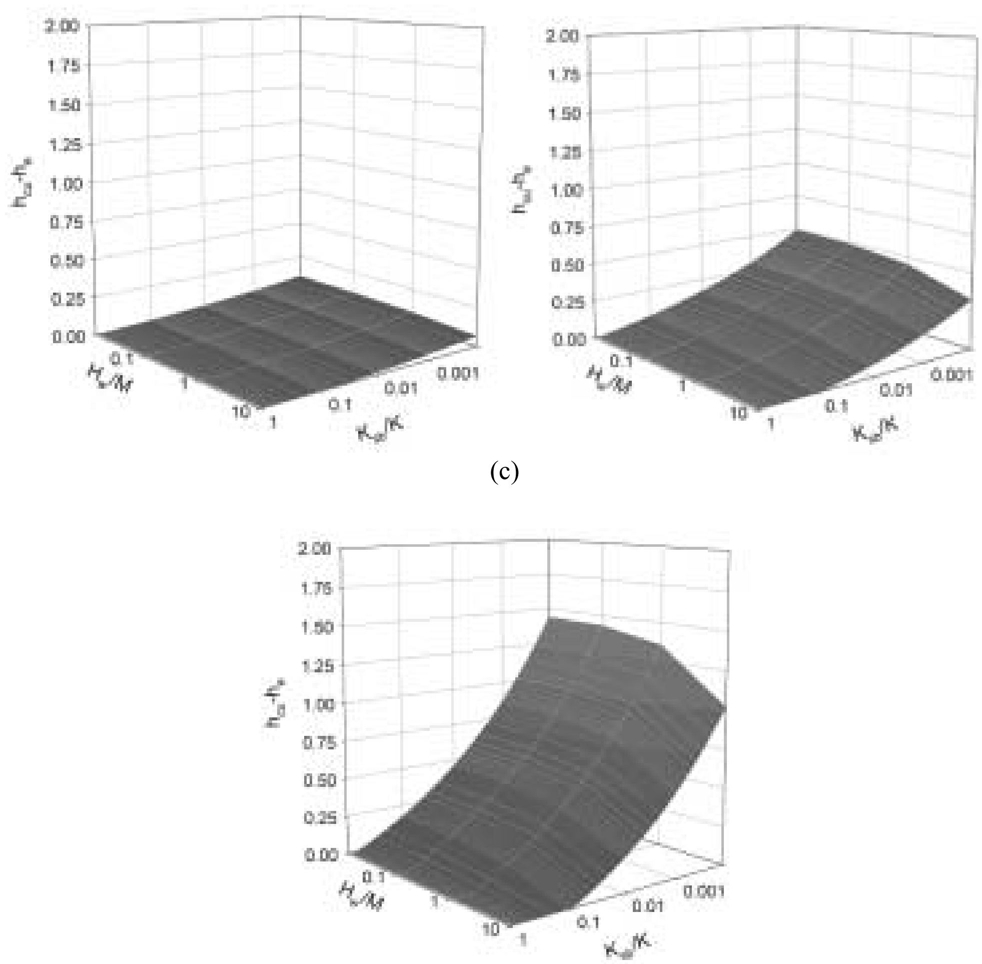

Figure 3. Difference between ultimate, h_{cu}, and entry, h_e, capillary pressure heads as a function of K_{sb}/K and H_w/M for (a) coarse sand (K=50 m-d^{-1}, h_e=5 cm), (b) medium sand (K=10 m-d^{-1}, h_e=20 cm), and (c) fine sand (K=2.5 m-d^{-1}, h_e=50 cm) subsoil with a streambed thickness, M=1 m.

Three different modifications of the current version of the MODFLOW RIVER package, RIV_M, are created for purposes of determining the influence of each regime on unsaturated stream/aquifer exchange. The four packages are summarized in Table 1. The first modified version, RIV_N, removes any consideration of unsaturated flow from the RIVER package. This version simulates the assumptions commonly addressed in analytical stream/aquifer solutions by assuming the stream remains a constant head boundary condition no matter how far the aquifer head drops below the bottom of the streambed. The next modified version is a RIVER package that simulates regime A (saturated flow), uses regime A to simulate the transition zone (regime B), and regime C (unsaturated flow), RIV_AC. In this scenario, saturated flow is assumed to occur until the drawdown falls a sufficient distance below the bottom of the streambed to form a maximum capillary pressure head (i.e., $s_w<H_w+M+h_{cu}$). This scenario is equivalent to the proposed modification of Osman and Bruen (2002). The final modified version of the RIVER package incorporates flow conditions governed by all three regimes, RIV_U.

Table 1. Summary of four MODFLOW RIVER packages used to investigate importance of unsaturated flow in stream/aquifer interaction.

RIV_M	Standard Version of MODFLOW RIVER Package	
		Saturated Flow: $s_w \le H_w+M$, $q \propto s_w$
		Unsaturated Flow: $s_w > H_w+M$, $q \propto H_w+M$
RIV_N	Only Saturated Flow	
		Saturated Flow: $s_w > 0$, $q \propto s_w$
		No Unsaturated Flow
RIV_AC	Accounts for Regimes A, B and C (Similar to Osman and Bruen, 2002)	
		Saturated Flow: $s_w \le H_w+M+h_{cu}$, $q \propto s_w$
		Unsaturated Flow: $s_w > H_w+M+h_{cu}$, $q \propto H_w+M+h_{cu}$
RIV_U	Accounts for Regimes A, B, and C	
		Saturated Flow: $s_w \le H_w+M+h_e$, $q \propto s_w$
		Transition Zone: $H_w+M+h_e > s_w > H_w+M+h_{cu}$, $q \propto H_w+M+h_{cl}$
		Unsaturated Flow: $s_w \ge H_w+M+h_{cu}$, $q \propto H_w+M+h_{cu}$

4. Significance of Stream/Aquifer Flow Regimes

The four MODFLOW RIVER packages listed in Table 1 are compared based on predicted drawdown and stream depletion for a coarse sand and fine sand aquifer. According to Larkin and Sharp (1992), representative aquifer systems for coarse sand river fill include the alluvial aquifers of the Arkansas River in Kansas, Great Miami River in Ohio, and South Platte River in Colorado. Representative aquifer systems for the fine sand river fill include the Brazos River in Texas, Red River in Louisiana, and the Colorado River in Texas.

A MODFLOW numerical model (Harbaugh and McDonald, 1996) is constructed to simulate a hypothetical system representing coarse and fine sand aquifers with a single pumping well (discharge rate, Q=10000 m^3-d^{-1}), located 125 m next to a partially penetrating stream. The alluvial aquifer is assumed homogenous, isotropic, and consisting of either coarse sand with a conductivity, K=50 m-d^{-1}, air entry capillary pressure head, h_e=5 cm and a pore size distribution index of 2 (i.e., η=8), or fine sand with K=5 m-d^{-1}, h_e=50 cm and η=5. The stream is assumed to have a width, W=5 m and a constant stream stage, H_w=0.5 m. The streambed is assumed to be homogeneous and isotropic, consisting of sandy loam and having a hydraulic conductivity, K_{sb}=0.5 m-d^{-1} and thickness, M=0.5 m.

The modified RIVER packages are independently integrated into the MODFLOW numerical model. Aquifer drawdown at the stream location nearest the pumping well and stream depletion is predicted by the four different RIVER packages. A comparison of predicted dimensionless drawdown (s_wT/Q, where T is the transmissivity of the aquifer) is shown in Figure 4 for the coarse sand aquifer and in Figure 5 for the fine sand aquifer. The ultimate capillary pressure head, h_{cu}, is approximately 8 cm for the coarse sand and 70 cm for fine sand aquifer.

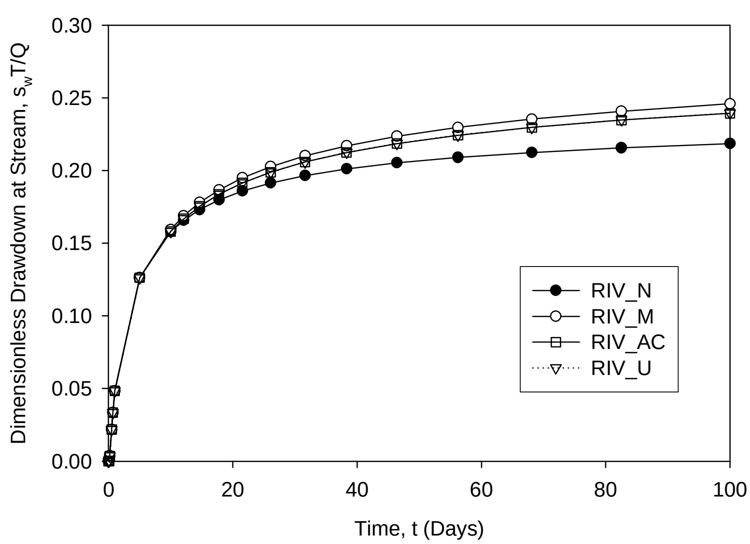

Figure 4. Comparison of dimensionless aquifer drawdown at the stream location nearest the pumping well as predicted by the four different MODFLOW RIVER packages for a fine sand aquifer (K=50.0 m-d^{-1}, h_e=5 cm).

RIV_M is the current RIVER package included in MODFLOW. RIV_M limits the amount of specific discharge to be proportional to the sum of the river stage and the streambed thickness. Therefore, RIV_M predicts a greater amount of drawdown and less stream depletion compared to RIV_N, RIV_AC and RIV_U. RIV_N allows the specific discharge through the streambed to increase with drawdown even after the water table has dropped below the streambed, resulting in more stream leakage to satisfy the aquifer stress due to pumping. RIV_AC and RIV_U predict minimal differences in aquifer drawdown and stream depletion, especially for the coarse sand aquifer. Small differences in predicted drawdown (i.e., less than 5 cm) are observed when comparing RIV_AC and RIV_U for the fine sand aquifer. Therefore, modeling regime B using the flow conditions of regime A may be appropriate for many stream/aquifer interaction scenarios.

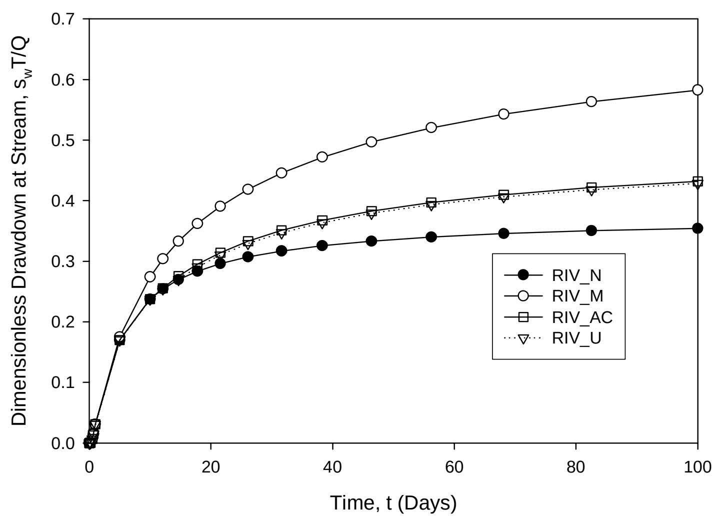

Figure 5. Comparison of dimensionless aquifer drawdown at the stream location nearest the pumping well as predicted by the four different MODFLOW RIVER packages for a fine sand aquifer (K=5.0 m-d^{-1}, h_e=50 cm).

5. Summary and Conclusions

Researchers are becoming aware of the importance of unsaturated flow in stream/aquifer exchange. Current models of stream/aquifer interaction include simplified analytical solutions that assume continual saturated flow between the stream and aquifer and more sophisticated numerical models that address the influence of unsaturated flow. However, the most widely used numerical groundwater-flow model, MODFLOW, uses inappropriate simplifying assumptions within its RIVER and STREAM packages when the aquifer head drops below the bottom of a semipervious streambed. This paper reviews the influence of unsaturated stream/aquifer interaction and attempts to improve the modeling capability of MODFLOW by improving its stream/aquifer interaction package. Also, the impact of different stream/aquifer flow regimes is demonstrated. The modified RIVER package presented in this research can account for saturated flow, a transition regime, and also unsaturated flow.

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