

## **Seasonal and regional variability in scaling properties and correlation structure of high resolution precipitation data in a highly heterogeneous mountain environment (Switzerland)**

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**Abstract.** A large dataset of high resolution 10-min precipitation data from 62 stations across Switzerland with an average of 21 years of observations are studied to explore the generality in the scaling relationships and correlation structure of precipitation. The focus is on the seasonal and regional variability in scaling and correlation parameters and their interdependency. It is shown that seasonal effects are generally stronger than regional ones. The summer season shows more structure in precipitation, a shorter autocorrelation range due to convective activity, high growth of intermittency and variability, and a resulting multiscaling behaviour in moments. Winter events are longer, with smoother, less variable, and strongly autocorrelated high resolution precipitation, and with a simple scaling behaviour caused by larger scale frontal events. Some coherent regional differences are also apparent. The high Alpine region shows less variability and a stronger autocorrelation than other regions, and a tendency towards simple scaling. It appears that orographic effects in the Alps lead to better behaved and more predictable precipitation fields. This paper shows that high resolution precipitation scaling and correlation parameters are considerably variable and interdependent. This has an important practical significance for the extrapolation of parameters of scaling-based models to ungauged sites.

### **1. Introduction**

The characterization of the scaling properties of precipitation is an important theoretical and practical issue in precipitation analysis and modelling (e.g., Rodriguez-Iturbe et al. 1998). Stochastic models of precipitation can be broadly generalized into models based on point process theory and those based on scale invariance. The latter take advantage of the fact that distribution properties of precipitation observed at different (space and time) resolutions appear to scale with the resolution and so the prediction of precipitation properties can be made across resolutions. Perhaps the most studied model in this group is the multiplicative random cascade which has evolved from the description of dissipation in turbulent flow (e.g., Kolmogorov 1962, Mandelbrot, 1974) and has led to many applications in multifractal rainfall analysis and modelling (e.g., Schertzer and Lovejoy 1987, Tessier et al. 1993, Harris et al. 1996, Svensson et al. 1996, Molnar and Burlando 2005, and others).

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The attractive feature of the multifractal approach to precipitation analysis is that it allows to describe the rather complex precipitation process over a wide range of scales with very few parameters. It is often implied that these parameters are fundamental descriptors of the physics of the precipitation process and a level of generality is attached to them. However, some case studies have shown that the parameters of the scaling behaviour and correlation structure may be substantially influenced by the nature of the climatic forcing (e.g., Svensson et al. 1996, Guntner et al. 2001, Kuzuha et al. 2004) and orography (e.g., Harris et al. 1996, Nykanen and Harris 2003). The generality of the scaling behaviour has also been questioned on theoretical grounds by Marani (2003) who showed that a single power law scaling in the variance cannot hold over all scales.

In this paper we address the generality of precipitation scaling and correlation parameters estimated from a large dataset of good quality time series of high resolution (10-min) precipitation data in Switzerland. We explore and illustrate the factors that influence the scaling relationships and correlation structure in precipitation. We focus in particular on the seasonal (winter/summer) and regional variability in the parameters and their interdependency. The Alpine mountain environment of Switzerland allows us to demonstrate some interesting effects of local climatology and orography, which may not be obvious in less heterogeneous environments.

## 2. Data

The data used in this study are 10-min precipitation records at selected 62 stations of the SMA MeteoSwiss network with an average of 21 years of observations (Figure 1). The data were verified with corrected hourly precipitation and inconsistencies were removed. The stations were divided into four climatologically meaningful regions.

The seasonal differences in precipitation in Switzerland are not large, except for the southern part of Switzerland where summer convective activity and autumn Mediterranean influence lead to more precipitation in these seasons than the rest of the year. Although the data presented in Figure 1 are not corrected for wind and exposure, it is obvious that altitude by itself is not a reliable predictor of mean annual (or seasonal) precipitation.

## 3. Methods

For all stations coarse-graining of 10-min precipitation data up to a scale of approximately 1-day was conducted, the moment scaling relationships were estimated on annual and seasonal bases, and intermittency, breakdown and correlation functions were parameterized at all scales.

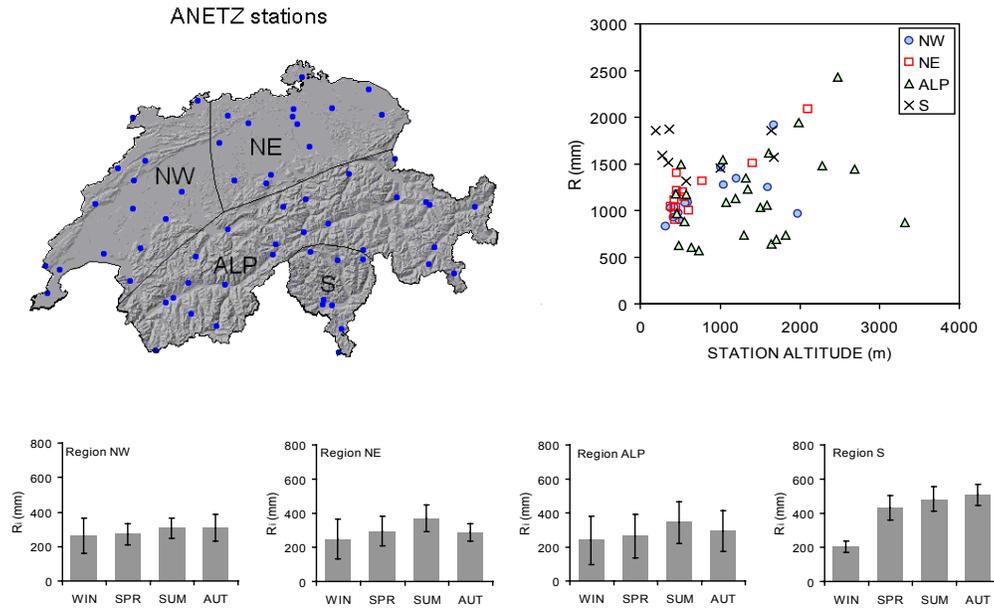
### 3.1. Variability and correlation structure

The 10-min precipitation data  $r(\tau)$  were used to estimate several basic statistics, among them the conditional variance ( $\text{Var}(r)$ ,  $r > 0$ ) and the mean event duration  $m$ , which was determined with an arbitrarily chosen separation interval of 3 hours between rainy periods.

Assuming that the data are second-order stationary on a seasonal basis, the correlation structure in the data was examined by fitting a correlation function (e.g., Menabde et al. 1997),

$$\langle r(\tau)r(\tau-t) \rangle \propto t^{-\mu} \quad (1)$$

to the 10-min data, where  $t$  is the lag-time and  $\mu$  is the correlation scaling exponent ( $\mu > 0$ ). The correlation scaling exponent is uniquely related to the Fourier transform of the power law spectrum. Values of the exponent  $\mu \approx 0$  indicate long-range correlation in the field. The power-law form of the correlation structure has been shown to be fundamental for reproducing the scaling structure in precipitation (e.g., Rodriguez-Iturbe et al. 1998, Marani 2003).



**Figure 1.** Location of 62 MeteoSwiss SMA/ANETZ stations with the division into the four studied climatological regions (top left). Uncorrected mean annual precipitation  $R$  as a function of station altitude (top right). Seasonal mean precipitation  $R_i$  by regions, together with bars denoting  $\pm 1$  standard deviation (bottom).

### 3.2. Scaling characteristics

The scaling analysis was conducted on non-overlapping doubling intervals from 10-min to 1280-min (21.3 hrs), to be compatible with the discrete random cascade model with branching number  $b = 2$ . The scaling range, from 10-min to approximately 1 day, is chosen in order to capture the most essential sub-daily scaling structure. In the cascade notation, the scale is defined as  $\lambda = 2^{-n}$ , where  $n$  is the level of the cascade development. For the 10-min data  $n = 7$  ( $\lambda = 0.0078$ ) for 1280-min data  $n = 0$  ( $\lambda = 1$ ).

The statistical moment  $M$  as a function of scale  $\lambda$  is defined as (e.g., Over and Gupta 1994, 1996),

$$M(\lambda, q) = \sum_i r(\lambda, i)^q \quad (2)$$

where the integral is over  $i$  intervals of aggregated rainfall  $r(\lambda, i)$  at scale  $\lambda$ , and  $q$  is the moment order. For a scaling field  $M$  behaves as,

$$M(\lambda, q) \propto \lambda^{-\tau(q)} \quad (3)$$

where the  $\tau(q)$  describes the nature of the scaling. If  $\tau(q)$  is a linear function of  $q$  then the field is simple scaling, if  $\tau(q)$  is a convex function of  $q$  then we call the field multiscaling (multifractal).

The  $\tau(q)$  function is related to the distribution of the cascade generator  $W$  (e.g., Tessier et al. 1993, Over and Gupta 1994). Because of its wide application, we illustrate here the parameters of the scaling behaviour in Eq. (3) assuming an intermittent lognormal model for the cascade generator (e.g., Over and Gupta 1994, 1996, Molnar and Burlando 2005),

$$\tau(q) = (\beta - 1)(q - 1) + \frac{\sigma^2 \ln 2}{2}(q^2 - q) \quad (4)$$

where  $\beta$  is a parameter that determines the growth of intermittency in the field ( $0 \leq \beta \leq 1$ ), and  $\sigma$  determines the variance of the cascade generator ( $\sigma \geq 0$ ). These two parameters are fundamental to the description of the scaling field. High  $\beta$  indicates high intermittency, the cascade generator may be zero with probability  $P(W = 0) = 1 - 2^{-\beta}$ . High  $\sigma$  indicates spikiness in the field and increased level of multiscaling (if  $\sigma = 0$  the field is simple scaling).

### 3.3. Breakdown distributions

The breakdown (or partition) distribution is the data-derived distribution of the cascade generator  $W$ , i.e. the ratio between the rainfall depth at two successive scales  $n$  and  $n + 1$  in a single subdivision. By definition,  $f(W)$  is bounded between 0 and 1, and may vary between scales.

We estimate  $f(W)$  by a symmetric Beta distribution  $B(a)$  with the parameter  $a$  inversely related to the variance  $\sigma_w^2$ ,

$$\hat{a} = 1/(8\sigma_w^2) - 0.5 \quad (5)$$

For  $a = 1$ ,  $W$  is uniformly distributed between 0 and 1, for  $a \gg 1$   $W$  is centered around the mean  $W = 0.5$  which leads to a less variable smoother field.

Data studies have shown that  $a$  depends on the scale  $\lambda$  (with decreasing resolution the field becomes smoother), and that bounded random cascades are needed to reproduce this behaviour (e.g., Menabde et al. 1997, Menabde and Sivapalan 2000, Molnar and Burlando 2005). Here we use the value of  $a$  estimated from the division between 10 and 20-min data, to illustrate the smoothness of the precipitation field at high resolutions.

## 4. Results

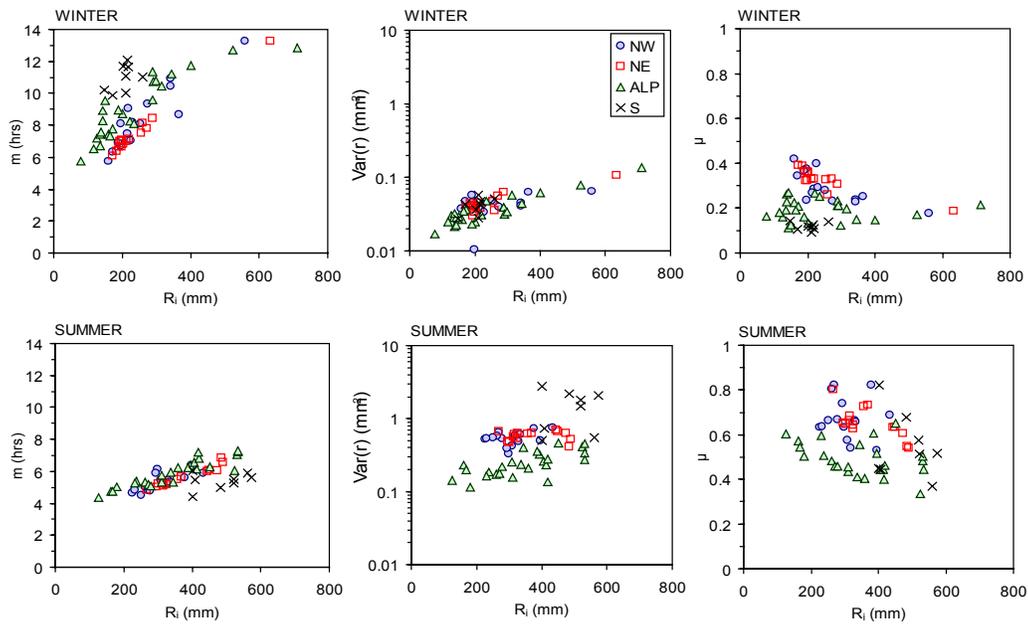
We investigate relationships between the parameters that describe the seasonal high resolution precipitation scaling ( $\beta$ ,  $\sigma$ ,  $a$ ) and correlation structure ( $\mu$ ) and measurement station characteristics such as location, altitude, mean annual precipitation, climatological region, etc. The focus in this paper is to contrast

the summer and winter seasons, to show the regional differences across Switzerland, and to illustrate the degree of dependence between the parameters.

#### 4.1. Seasonal variability

Seasonal variability is a strong component of the variability in all precipitation characteristics.

A comparison of the mean event duration, variance and the correlation scaling exponent for winter and summer seasons shows that in the winter, mean event duration is longer, the variability is substantially smaller, and the autocorrelation is stronger (Figure 2). This is mostly due to frontal storms that are prevalent in the winter as opposed to summer convective activity. Short duration summer storms lead on the average to a higher variability in rainfall and also a decrease in the temporal correlation range.

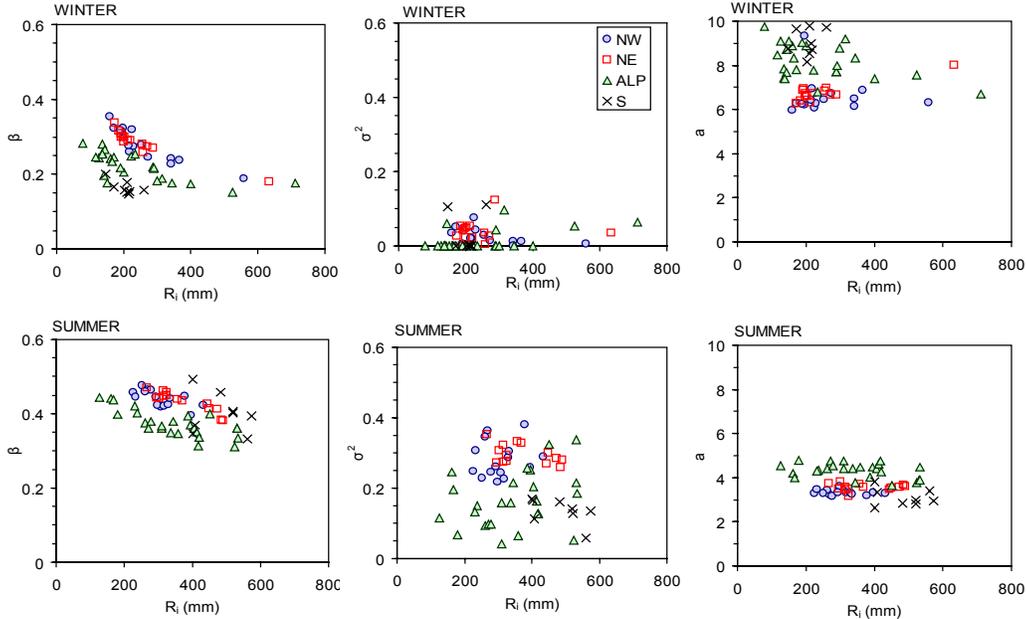


**Figure 2.** Mean event duration  $m$  (left), conditional variance of 10-min data (center), and the correlation scaling exponent  $\mu$  (right) for the winter (top row) and summer (bottom row) seasons for the four climatological regions. Data are plotted as a function of seasonal mean precipitation at each station  $R_i$ .

The scaling structure of precipitation also shows large differences between winter and summer seasons (Figure 3). In winter, intermittency ( $\beta$ ) and spikiness (the variance of the cascade generator  $\sigma$ ) are significantly lower than in the summer. The structure of the precipitation fields in winter is close to simple scaling ( $\sigma \approx 0$ ), while in summer it is multiscaling in all cases ( $\sigma > 0$ ). The negative relation between  $\beta$  and mean seasonal precipitation  $R_i$  is a large scale forcing dependency that has been used in simulation (e.g., Over and Gupta 1996, Molnar and Burlando 2005).

The differences between the winter and summer breakdown distribution parameters  $a$  are statistically most significant (Figure 3). Winter events are

smoother at the high resolution, that is the distribution of the breakdown coefficients  $W$  estimated for the 10-20-min partition has a much lower variance  $\sigma_W^2$  (therefore larger  $a$ ) than for summer events. The parameter  $a$  converges to a uniform distribution at scales above 640-min for both seasons. This indicates that bounded multiplicative random cascades should be a better descriptor of the precipitation process in the scaling range studied here.



**Figure 3.** Precipitation scaling parameters: intermittency  $\beta$  (left), variance of the generator  $\sigma^2$  (center), breakdown distribution parameter  $a$  for the 10-20 min partition (right) for the winter (top row) and summer (bottom row) seasons for the four climatological regions. Data are plotted as a function of seasonal mean precipitation at each station  $R_i$ .

## 4.2. Regional variability

Although seasonal variability in parameters is the strongest signal we found, there are also indications of important regional variability. Two regions in particular exhibit some interesting deviations: the high Alps and southern Switzerland (Tessin).

For example, winter events are generally longer in southern Switzerland than in other regions with the same seasonal precipitation total, while summer convective rainfall events are generally shorter but highly variable (Figure 2). The high Alpine region generally shows less variability and a stronger autocorrelation than other regions with the same seasonal precipitation total, even though mean event duration is approximately the same.

The scaling parameters support this picture (Figure 3). Southern Switzerland shows low intermittency and high smoothness in winter, while the opposite is true for summer. The interesting deviation in the high Alps is in the summer, when low intermittency combined with a strong auto-correlation leads to precipitation events that have a lower variability and a simple scaling structure compared to other regions with the same seasonal precipitation total.

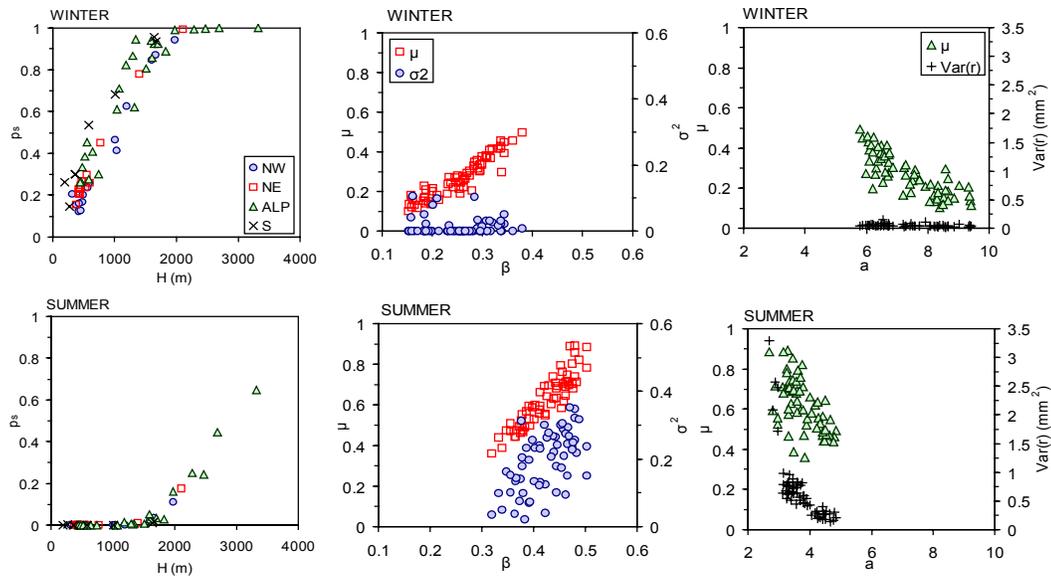
One could conclude that orographic effects in the Alps lead to better behaved and more predictable precipitation fields than in the prealpine regions.

### 4.3. Dependence between parameters

It is evident from the above results, that some of the precipitation parameters studied here cannot be independent. Although intuitively obvious, the relationships between scaling parameters in precipitation are not commonly reported in the literature.

The strongest positive correlation was found between the correlation scaling exponent  $\mu$  and the intermittency parameter  $\beta$  (Figure 4). This is because with high growth of intermittency the correlation structure in the precipitation field is destroyed. The correlation is better in the winter than in the summer season. When multiscaling appears, there is a positive correlation between the variance of the cascade generator  $\sigma$  and the intermittency parameter  $\beta$ .

The smoothness of the high resolution precipitation field signified by the breakdown distribution parameter  $a$  is related to the exponent  $\mu$  and to the variance of precipitation. With increasing smoothness (high  $a$ ), the correlation range increases (low  $\mu$ ), while the variance decreases. This is true in both seasons, but especially in the summer. This (and other) dependencies between scaling parameters should be accounted for when stochastic precipitation models are being calibrated.



**Figure 4.** The relative frequency of snow  $p_s$  as a function of station altitude (left). Scatterplots of the correlation parameter  $\mu$  and scaling parameter  $\sigma^2$  as a function of intermittency  $\beta$  (center), and of  $\mu$  and the conditional variance of 10-min data as a function of the breakdown distribution parameter  $a$  for the 10-20 min partition (right) for the winter season (top row) and summer (bottom row) seasons.

None of the parameters examined here had statistically significant relationships with station altitude. However, we think that one of the main reasons for the variability in the winter season parameters between regions is due to the presence of snowfall. To illustrate the possible importance of snowfall, we estimated the relative frequency of snowfall  $p_s$  as,

$$p_s = n_T / n_{r_h} \quad (6)$$

where  $n_T$  is the number of hourly intervals with measured precipitation with mean air temperature  $T < 1^\circ\text{C}$  and  $n_{r_h}$  is the number of hourly precipitation intervals with  $r_h > 0.3$  mm. Figure 4 shows that  $p_s$  is strongly dependent on altitude and less on the region. Notably  $p_s > 0$  for the winter season at all stations, and for altitudes  $H > 2000$  m, almost all precipitation falls as snow. We believe the question whether the scaling structure of snowfall events is different from rainfall is an important one, and we are not aware of research efforts that have examined this in sufficient detail so far.

## 5. Conclusions

A large dataset of high resolution (10-min) precipitation data at 62 stations across Switzerland with an average of 21 years of observations is used to explore the generality of the scaling relationships and correlation structure of precipitation. We focused in particular on the seasonal and regional variability in parameters and their interdependency.

Seasonal effects on scaling and correlation parameters are generally stronger than regional ones. The summer season generally shows more structure in precipitation, a shorter autocorrelation range due to convective activity, high growth of intermittency and variability, and a resulting multiscaling behaviour in moments. Winter events are longer, with smoother, less variable, and strongly autocorrelated high resolution precipitation, and with a simple scaling behaviour caused by larger scale frontal events. These results are in general agreement with previous work (e.g., Kuzuha et al. 2004)

Although smaller than the seasonal effects, coherent and sometime strong regional differences are apparent. Most obvious and evident are the differences in the high Alpine and southern Switzerland regions. In southern Switzerland, winter events are generally longer, smoother and less intermittent than in other regions with the same seasonal precipitation total, while the opposite is true in summer. The high Alpine region exhibits less variability and a stronger autocorrelation than other regions, even though mean event duration is approximately the same. In summer, low intermittency combined with a strong autocorrelation leads to precipitation events that have a lower variability and simple scaling structure compared to other regions. It appears that orographic effects in the Alps lead to better behaved and more predictable precipitation fields.

Results show that some of the parameters are strongly dependent. For example, strong correlations were found between the correlation scaling exponent, the variance of the cascade generator, and intermittency. The smoothness of the high resolution precipitation field signified by the breakdown distribution parameter is related to the correlation range and variance. Such dependen-

cies between scaling parameters should be accounted for when stochastic precipitation models are being calibrated.

The analysis of precipitation scaling and correlation parameters in this paper shows that parameter variability may be large and depend to a substantial degree on season and local climatology. This has an important practical significance for the extrapolation of parameters of scaling-based models to ungauged sites.

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