Application of Fuzzy Set Theory to Flood Risk Analysis in Natural Rivers as a Function of Hydraulic Parameters

Silvia Helena Santos[2],Patrícia Freire Chagas[2], Carla Freitas Andrade[1], Vanessa Ueta[2] and Raimundo Souza[2]
Universidade Federal do Ceará, Fortaleza, Ceará, Brazil

Abstract. A mathematical model based on the Saint-Venant hydrodynamic equations, combined with fuzzy set theory, is developed in order to study the influence of hydraulic parameter of Natural River on the flood risk. To do so, the set of differential equations, in FUZZY formulation, is solved by numerical difference method, so that the membership functions of all control variables related with the flow could be calculated. With base in these membership functions the model is capable of evaluating the fuzzy risk for such area subjected to the flood process during intense rains. A computer program QUARIGUA (Risk Quantitative Analysis of Flooding in Urban Rivers) is used to perform the simulations. The computer program QUARIGUA is organized in a modular manner, with two main modules: the deterministic module, where the depth of the water in the river and the flow of the channel are calculated as discreet values; and the fuzzy module, based on the fuzzy set theory, where the depth of the water and the flow are calculated as membership functions. The simulations demonstrate the reliability, versatility and computational efficiency of the proposed model to evaluate fields of risk and reliability in hydrodynamics systems.
Keywords: Flood Control; Hydrodynamic Models; Risk Analysis.

1. Introduction

To begin the studying of urban planning close to any natural river, it is necessary to know the dynamics of the river. In other words, it is important to know its behavior, considering that the propagation of a flood wave, in the space and in the time, it is a complex problem.

Usually, the mathematical models used to describe the unsteady flow in open channels are composed by the equations of the continuity and momentum conservation, developed by Saint-Venant, that are partial differential equations, strongly no lineal, whose solution can be obtained using some numerical scheme available in the literature.

However, considering that this studying is pertinent to a flood risk analysis, there is a more necessary evaluation of the present uncertainties in the flow processes. Such analysis can be developed through the use of the theory probability theory, or the fuzzy set theory or yet through the combination of these two theories. As the probabilistic theory demands a solid database for its analysis, that most of time is not available to be used, the present work was using of the fuzzy set theory, whose main advantage is not to need of great groups of data to reach their main objectives.

Therefore, this research is presenting fuzzy theory applied to the hydrodynamic model ir order to evaluate the flood risk analysis in natural rivers. The model solves the equations of Saint-Venant using the difference finite method, with an implicit scheme, proposed by

[1] Professor, Departamento de Engenharia Mecânica e de Produção, Universidade Federal do Ceará, Fortaleza, Ceará, Brazil. Email: carla@ufc.br
[2] Professor, Departamento de Engenharia Hidráulica e Ambiental, Universidade Federal do Ceará, Fortaleza, Ceará, Brazil. Email: rsouza@ufc.br

Chow (1988). The results have shown that the proposal methodology could become a good alternative to the study all king of problem related with engineering risk analysis.

2. Methodology Proposed

The methodology that is developed to these studying concerns of two parts: first the flow must be calculated in such way that all control variable must appear in membership functions form. Second these results must be used to calculate the field of flood risk for every section and every time step. The flow field, in the river, is obtained through the numeric solution of the Saint-Venant equations. Those equations, of the continuity and of the momentum, are described, according to Chow (1988):

- Continuity Equation

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \tag{1}$$

- Momentum Equation

$$\frac{\partial Q}{\partial t}+\frac{\partial\left(Q^2/A\right)}{\partial x}+gA(\frac{\partial y}{\partial x}-S_0)+gAS_f=0 \tag{2}$$

where x is the longitudinal distance along the channel (m), t is the time (s), A is the cross section area of the flow (m^2), y is the surface level of the water in the channel (m), S_0 is the slope of bottom of the channel, S_f is the slope of energy grade line, B is the width of the channel (m), and g is the acceleration of the gravity ($m.s^{-2}$).

In order to calculate S_f, the Manning formulation will be used. Thus,

$$V=\frac{1}{n}R^{2/3}S_f^{\ 1/2} \tag{3}$$

where V is the mean velocity (m/s), R is the hydraulic radius (m) e n is the roughness coefficient.

- Initial Conditions

$$Q(x,0)=Q_0 \tag{4}$$

$$A(x,0)=A_0 \tag{5}$$

where Q_0 is the steady state flow of the channel, and the A_0 is the cross section area for the steady state conditions.

- Boundary Conditions:

$$Q(0,t) = f(t) \tag{6}$$

where *f(t)* is the hydrograph.

3. Fuzzy Set Theory

One way to evaluate the risk of collapse of any system is through the Fuzzy Set Theory. This theory, appeared in the sixties, it represents an important tool in the science or technology problem concerning with high degree of uncertainties. In this case, the theory in subject develops an important game, mainly, because its application does not demand a rigorous database.

In the field of water resources engineering, especially, in the subjects of water resources planning, the application of fuzzy number theory is beginning growing. At this time, there are not works in the literature that treat with this methodology. However, some works that were already developed show the great potential that this theory to investigate all problem concerning with flood risk analysis.

The concept of uncertain or fuzzy number may be presented in many ways. As point out Vieira (2005), the fuzzy number can be defined as an extension of the concept of an interval of confidence. This extension is based on a natural and very simple idea. Instead of considering the interval of confidence at one unique level, it is considered as a several levels from 0 to 1. Mathematically speaking, one can define fuzzy number as follow.

Let F be a referential set. An ordinary subset A. of this referential set is defined by it characteristic function, called the membership function, which takes its values in the interval [0,1]. In such way, for any x, belonging to F, $\mu_A(x) \in F$, that is, the elements of F belongs to A with a level located in [0,1].

A pair of equations defined into some interval can represent a membership function. For example, a membership function of a fuzzy number may be described mathematically by means of two strictly function L and R defined by;

$$\mu_x(x) = L(\frac{x_m - x}{x_1}), \quad x < x_m, \quad x_1 > 0 \tag{7}$$

$$\mu_x(x) = R(\frac{x - x_m}{x_2}), \quad x > x_m, \quad x_2 > 0 \tag{8}$$

where, x_1, x_2 and x_m are number with defined level or confidence.

The application of that theory on the hydrodynamic equation brings the fuzzy hydrodynamics modeling that was proposed in this research. Therefore, applying the fuzzy theory into the equations (1-6), the new formulation for the model is represented below through the formulation,

- Continuity Fuzzy Equation

$$\frac{\partial \widetilde{A}}{\partial t} + \frac{\partial \widetilde{Q}}{\partial x} = \widetilde{q} \tag{9}$$

- Momentum Fuzzy Equation

$$\frac{1}{\tilde{A}}\frac{\partial \tilde{Q}}{\partial t}+\frac{1}{\tilde{A}}\frac{\partial}{\partial x}\left(\frac{\tilde{Q}^2}{\tilde{A}}\right)+g\frac{\partial \tilde{y}}{\partial x}-g\left(\tilde{S}_0-\tilde{S}_f\right)=0 \tag{10}$$

where $\tilde{A}$ is the membership function for the transversal area of the river; $\tilde{Q}$ is the membership function for the flow; $\tilde{y}$ is the membership function for the depth; $\tilde{q}$ is the membership function for lateral flow; $\tilde{S}_0$ is the membership function for the bed slope of the river; and $\tilde{S}_f$ is the membership function for the headline slope.

- Boundary Conditions:

$$\tilde{Q}(0,t)=\tilde{Q}_0(t) \tag{11}$$

$$\left.\frac{\partial \tilde{Q}}{\partial x}\right/_{x=L}=0 \tag{12}$$

- Initial Conditions:

$$\tilde{Q}(x,0)=\tilde{Q}_1(x) \tag{13}$$

- Energy Gradeline equation

$$\tilde{Q}=\frac{1}{\tilde{n}}\tilde{A}\tilde{R}^{2/3}\tilde{S}_f^{\,1/2} \tag{14}$$

This is the new model in its fuzzy form. It is important to remind that the solution of this model will supply four membership functions, one for each control variable. Therefore, a membership function will exist for the flow, the velocity, the flow depth and the cross section area of the channel. These membership functions can be used in the evaluation of the flood risk analysis.

4. Results

The hydrodynamic fuzzy model developed in this work was used to accomplish several simulations, where the bed slope and roughness coefficient were changed, for the same initial and boundary conditions, in way to evaluate the propagation of the fuzzy risk wave, along the channel.

In figures 1 and 2, one can see the results of a simulation where it was just varied the amplitude of the flood wave. In this case, it is possible to see three sceneries with picks of waves 3, 4 and 5 larger times than the normal flow, respectively. The results show, clearly, the consistence of the used methodology where as risk wave became bigger as bigger is the pick of the wave. In the same way, the reliability wave has inverse behavior concerns with the risk behavior. On the other hand, as bigger is the flood wave amplitude as smaller will be the reliability amplitude.

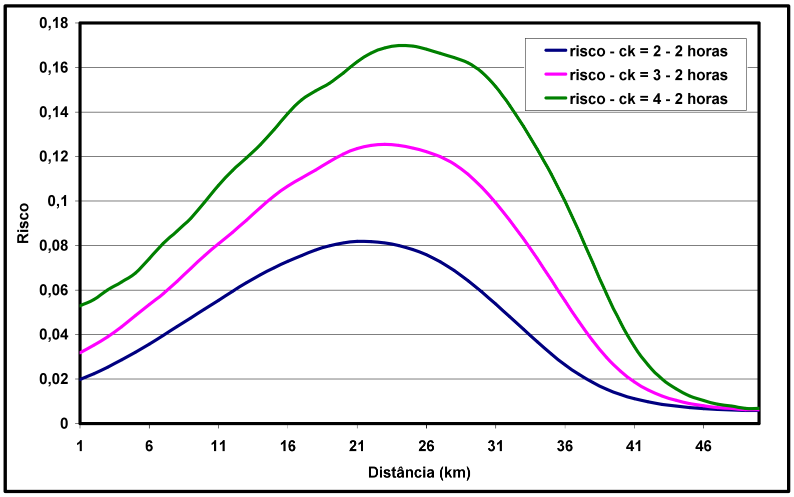

Figure 1. Fields of flood risk behavior for different flood wave amplitude.

That is explained, considering that, as bigger is the flow pick, for the same roughness coefficient scenery and bed slope, the bigger will be the wave generated by the behavior of the depth flow, making the risk increases and the reliability decreases. It is important to pay attention to the transient aspect of these functional set that change for section to section and from time to time, in function of the behavior of the hydrologic of the basin, or in function of the flow behavior.

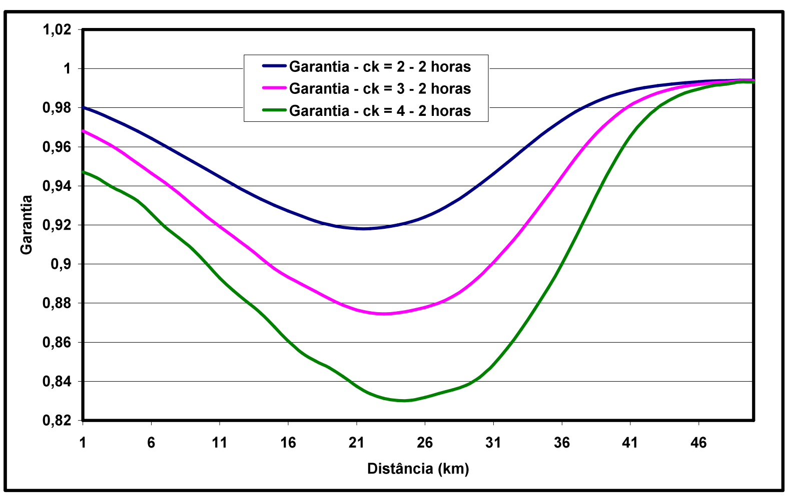

Figure 2. Field of reliability behavior, for different flood wave amplitude.

The Figure 3 compares the results of the risk for different roughness values, with time, allowing, like this, to prove the previous analysis. Increasing its value makes the risk bigger. When the roughness coefficient passes from 0,01 to 0,03, the risk increases significantly. These results are very important to show the sensibility of risk with that parameter.

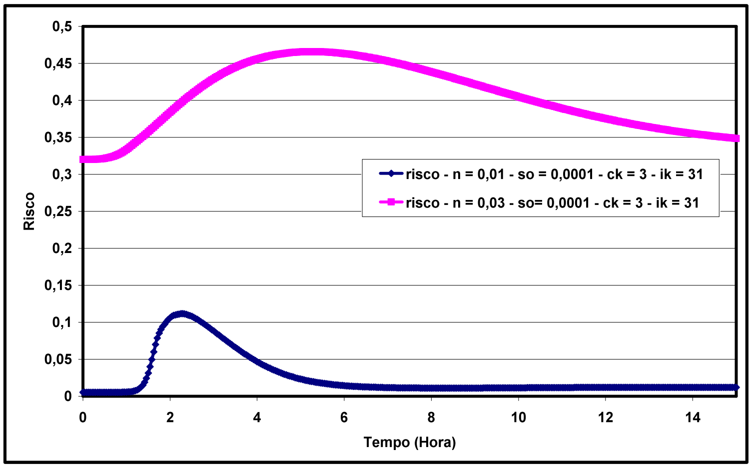

Figure 3. Flood risk behavior for different roughness at 30 Km from the beginning.

Regarding the bed slope, some simulations were also accomplished for two different membership functions, where their values are [0.00075; 0.0001; 0.00125] and [0.0000375; 0.00005; 0.0000625]. The results allow concluding that the bed slope plays a part equally important in the behavior of the risk. Through the figure 4, it can be verified that, as bigger is the bed slope, the smaller will be the flood risky. These results obey, as in the previous analysis, to the behavior of the bed slope of the main channel.

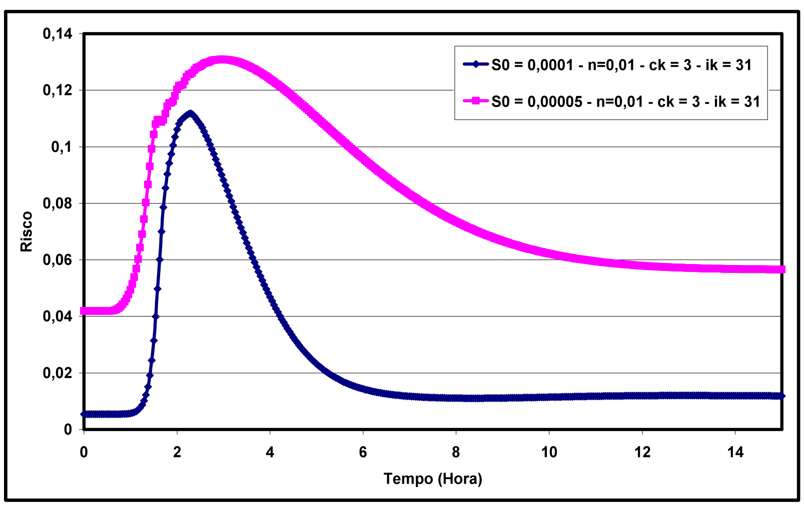

Figure 4. Flood risk behavior for different bed slope at 30 Km from the beginning.

5. Conclusions

After a series of simulations to represent different practical situations, accomplished with the application of the computational mathematical model QUARIGUA, especially developed for this research, the found results allow the following conclusions to be reached.

Regarding the calculation of the risk, the proposed mathematical model showed, through its solution, a significant influence in the distribution and quantification of this function. The computational program developed allowed to establish the fields of flood risk, along the channel, for different times, when of the passage of a flood wave. With that, it was possible to verify that there is propagation in the risk field that is developing with the same frequency of the propagation of the flood wave. Such a result allowed concluding that it is possible, through that methodology, to calculate the distribution of the flood risk, along the channel, for different times, in function of flood wave;

It was made an analysis of the influence of the bed slope of the channel and of the roughness coefficient, in the evaluation of the flood risk. The results showed that, as it was expecting, as bigger is the roughness coefficient, the bigger will be the flood risk and the smaller will be the reliability, in the whole extension of the channel. This same result is valid when the evolution of the flood risk is evaluated in a section of the channel. For instance, being taken a section located at 30 Km from the origin, and considering the same entrance data, just varying the roughness coefficient of 0.01 to 0.03, in their values of maximum degree of confidence. In such situation, the maximum risk in the referred section, passed from 11% to 46%, that it proves the previous analysis;

Similar analysis was made regarding the bed slope of the channel. The results showed that, under the same conditions, when the bed slope changed from 0.0001 to 0.00005, the maximum pick of the risk jumped from 11% to 13%. However, in spite of this result not to be so significant, the results showed that the risk is stabilized, for this different bed slope, with a considerable difference. For instance, when this parameter is 0.0001, the risk was stabilized close to 1%, and when the bed slope goes to 0.00005, the flood risk was stabilized in 6%. However, it can be concluded that the influence of the roughness coefficient in the flood risk is more significant than the influence of the bed slope.

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